**Math 100C: Spring 2017**
**Midterm 1**

**Instructions:** This is a 50 minute exam. No books, notes, electronic devices, or interpersonal assistance are allowed. Write your answers in your blue book, making it clear what problem you are working on. Be sure to justify your answers. This exam is out of 100 points; you get 5 points for legibly writing your name on your blue book. Good luck!

**Problem 1:** [5 + 15 pts.] Let $G$ be a group and let $R : G \to GL_n(\mathbb{C})$ be a complex matrix representation of $G$. Let $V = \mathbb{C}^n$ be the associated $G$-module.
(a) Carefully state what it means for $V$ to be "irreducible".
(b) Let $R : \mathbb{Z} \to GL_2(\mathbb{C})$ be the matrix representation defined by

$$R_n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}.$$

Is the associated $G$-module $V = \mathbb{C}^2$ irreducible? Justify your answer.

**Problem 2:** [25 pts.] Give an example of a finite group $G$, an irreducible $G$-module $V$ *over field* $\mathbb{R}$ *of real numbers*, and a $G$-module homomorphism $\varphi : V \to V$ which is not multiplication by a scalar. Make sure it is clear why your example works.

**Problem 3:** [25 pts.] Let $D_4 = \langle r, s \mid r^4 = s^2 = 1, srs = r^{-1} \rangle$ be the dihedral group of symmetries of the square. Calculate the character table of $D_4$.

**Problem 4:** [10 + 15 pts.] Let $R : S_3 \to GL_2(\mathbb{C})$ be the 2-dimensional irreducible matrix representation of the symmetric group $S_3$. Define a new 4-dimensional representation $R' : S_3 \to GL_4(\mathbb{C})$ by

$$R'_\sigma := R_\sigma \otimes R_\sigma \qquad \text{(matrix tensor product)}$$

for all permutations $\sigma \in S_3$ (you need not prove that $R'$ is actually a representation – this operation is known as *Kronecker product*). Let $\chi_{R'} : S_3 \to \mathbb{C}$ be the character of $R'$.
(a) Calculate the value of $\chi_{R'}$ on each of the three conjugacy classes in $S_3$.
(b) Write $\chi_{R'}$ as a sum of irreducible $S_3$-characters.