Math 100C: Spring 2017
Practice Midterm 1

Instructions: This is a 50 minute exam. No books, notes, electronic devices, or interpersonal assistance are allowed. Write your answers in your blue book, making it clear what problem you are working on. Be sure to justify your answers. This exam is out of 100 points; you get 5 points for legibly writing your name on your blue book. Good luck!

Problem 1: [5 + 15 pts.] Let G be a group and let V be a nonzero G-module over some field k.

(a) Carefully state what it means for V to be "indecomposable".

(b) Suppose V is irreducible. Prove that V is indecomposable.

Problem 2: [25 pts.] Let G be a finite group and let V be a finite-dimensional G-module over $\mathbb{C}$. The *Reynolds operator* $R : V \to V$ is defined by

$$R(v) := \frac{1}{|G|} \sum_{g \in G} g.v,$$

for all $v \in V$. Prove that R projects V onto the subspace $V^G \subseteq V$ of G-invariants:

$$V^G := \{v \in V \ : \ g.v = v \text{ for all } g \in G\}.$$

Problem 3: [25 pts.] Write down the character table of the product group $S_3 \times S_2$.

Problem 4: [25 pts.] Consider the additive group of integers $G = \mathbb{Z}$. Give an example of two complex matrix representations

$$R, S : \mathbb{Z} \to GL_2(\mathbb{C})$$

of $\mathbb{Z}$ of dimension 2 such that we have an equality of characters

$$\chi_R = \chi_S,$$

but such that R and S are *not* isomorphic.