Math 100C: Spring 2017
Practice Midterm 2

Instructions: This is a 50 minute exam. No books, notes, electronic devices, or interpersonal assistance are allowed. Write your answers in your blue book, making it clear what problem you are working on. Be sure to justify your answers. This exam is out of 100 points; you get 5 points for legibly writing your name on your blue book. Good luck!

Problem 1: [5 + 20 pts.] Let $K/F$ be an extension of fields.
(a) Carefully state what it means for the extension $K/F$ to be a ‘splitting field over $F$’.
(b) Let $\alpha = 2^{1/4}$ be the real fourth root of 2. Is the extension $\mathbb{Q}(\alpha)/\mathbb{Q}$ a splitting field over $\mathbb{Q}$? Justify your answer.

Problem 2: [10+15 pts.] (a) Prove that the polynomial
$$f(x) = x^4 + x^3 + x^2 + x + 1 \in \mathbb{Q}[x]$$
is irreducible over $\mathbb{Q}$.
(b) Let $K$ be the splitting field of the polynomial $g(x) = x^5 - 3$ over $\mathbb{Q}$. Determine the degree $[K : \mathbb{Q}]$.

Problem 3: [10 + 10 pts.] Let $F$ be a field of order 32.
(a) How many fields of order 8 does $F$ contain? Justify your answer.
(b) To what product of cyclic groups is the additive group $F^+$ isomorphic? Justify your answer.

Problem 4: [10 + 15 pts.] (a) Let $K/F$ be a finite extension of fields. Prove that $K/F$ is algebraic.
(b) Let $F \subseteq L \subseteq K$ be a tower of fields. Suppose $K/L$ and $L/F$ are algebraic. Prove that $K/F$ is algebraic.