Problem 1: (Exercise 9.6 in Gallian) Let \( H = \left\{ \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \mathbb{R} \right\} \). Is \( H \) a normal subgroup of \( GL(2, \mathbb{R}) \)?

Problem 2: (Exercise 9.22 in Gallian) Determine the order of \( (\mathbb{Z} \oplus \mathbb{Z})/\langle (2, 2) \rangle \). Is this group cyclic?

Problem 3: Give an example of a group \( G \) and subgroups \( K < H < G \) such that \( K \triangleleft H \) and \( H \triangleleft G \) but \( K \) is not a normal subgroup of \( G \).

Problem 4: (Exercise 9.40 in Gallian) Let \( \phi : G \rightarrow H \) be an isomorphism of groups. Suppose that \( K \) is a normal subgroup of \( G \). Prove that \( \phi(K) \) is a normal subgroup of \( H \).

Problem 5: (Exercise 9.62 in Gallian) Let \( G \) be a group and let \( G' \) be the subgroup of \( G \) generated by the set of all elements of the form \( xyx^{-1}y^{-1} \), where \( x, y \in G \). (\( G' \) is called the \textit{commutator subgroup} of \( G \).)

1. Prove \( G' \) is normal in \( G \).
2. Prove \( G/G' \) is Abelian.
3. If \( N \triangleleft G \) and \( G/N \) is Abelian, prove that \( G' \leq N \).
4. Prove that if \( G' \leq H \leq G \), then \( H \) is normal in \( G \).

Problem 6: (Exercise 10.14 in Gallian) Prove that the mapping \( \phi : \mathbb{Z}_{12} \rightarrow \mathbb{Z}_{10} \) given by \( \phi(x) = 3x \) is not a homomorphism.

Problem 7: If \( \phi : G \rightarrow H \) and \( \psi : H \rightarrow K \) are group homomorphisms, prove that \( \psi \circ \phi : G \rightarrow K \) is also a homomorphism.

Problem 8: (Exercise 10.26 in Gallian) Determine all homomorphisms from \( \mathbb{Z}_4 \) to \( \mathbb{Z}_2 \oplus \mathbb{Z}_2 \).

Problem 9: Let \( G \) and \( H \) be groups. Prove that the projection map \( \pi : G \oplus H \rightarrow G \) given by \( \pi(g, h) = g \) is a group homomorphism. Deduce that \( \{e\} \oplus H = \{(e, h) : h \in H\} \) is a normal subgroup of \( G \oplus H \) and \( (G \oplus H) / \{(e) \oplus H\} \approx G \).