**Problem 1:** Let $G = \mathbb{Z}_2 \oplus \mathbb{Z}_2$. Then $G$ is Abelian, but $G$ does not contain an element of order 4 (indeed $|(0,0)| = 1$ and $|(1,0)| = |(0,1)| = |(1,1)| = 2$). Since $|G| = 4$, $G$ is not cyclic.

**Problem 2:** We know that $|g|$ divides $|G| = 16$. Since $g^8 \neq e$, we also know that $|g|$ does not divide 8. This forces $|g| = 16$, which implies that $|g^2| = 8$.

**Problem 3:** We know that $|A_4| = \frac{4!}{2} = 12$ and that $\langle (1,2,3) \rangle$ has order 3. Therefore, the index of $\langle (1,2,3) \rangle$ in $A_4$ is $\frac{12}{3} = 4$, so $\langle (1,2,3) \rangle$ has 4 distinct right cosets in $A_4$.

**Problem 4:** The order of a permutation written in disjoint cycle notation is the least common multiple of its cycle lengths. Therefore, $(1,2,3)(4,5)$ has order 6 in $S_5$. We also know that $R_{90}$ has order 4 in $D_4$. The order of $\langle (1,2,3)(4,5), R_{90} \rangle$ in $S_5 \oplus D_4$ is therefore $\text{lcm}(6,4) = 12$.

**Problem 5:** Suppose $\text{Inn}(G)$ has order 1. Let $x, y \in G$. We know that the map $g \mapsto xgx^{-1}$ is the identity function on $G$. In particular, $y = xyx^{-1}$, or $yx = xy$. Therefore, $G$ is Abelian.

Suppose $G$ is Abelian. Let $x \in G$. For any $g \in G$, $xgx^{-1} = gxg^{-1} = g$, so the inner automorphism $\phi_g : G \rightarrow G$ given by $\phi_g(x) = gxg^{-1}$ is the identity function on $G$. It follows that $\text{Inn}(G)$ has order 1.

**Problem 6:** We claim that such a list is given by $\mathbb{Z}_{12}, \mathbb{Z}_6 \oplus \mathbb{Z}_2, A_4, D_6$. The Abelian groups $\mathbb{Z}_{12}$ and $\mathbb{Z}_6 \oplus \mathbb{Z}_2$ are not isomorphic because $\gcd(2,6) \neq 1$. No Abelian group is isomorphic to a non-Abelian group, so it is enough to show that $A_4$ and $D_6$ are not isomorphic. Indeed, $A_4$ contains only elements of orders 1, 2, and 3 whereas $D_6$ contains an element of order 6 (namely, rotation of the regular hexagon by one notch).