Math 104A: Fall 2012
Homework 2
Due Friday, 10/12/2012 at 5:00 pm

1. Define the Fibonacci numbers F_n by $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n > 2$. Prove that no two successive Fibonacci numbers F_n and F_{n+1} have a common divisor $a > 1$.

2. Prove that $\sum_{m=1}^{n} m^2 = \frac{1}{6}n(n+1)(2n+1)$.

3. Let R be an ordered ring such that R^+ is well ordered. A subset $T \subseteq R$ is said to be bounded below if there exists $m \in R$ such that $m \leq t$ for all $t \in T$. Show that if T is nonempty and bounded below, then T contains a (unique) minimum element. (That is, there exists $t_0 \in T$ such that $t_0 \leq t$ for all $t \in T$.)

4. Let R be an ordered ring such that R^+ is well ordered. A subset $S \subseteq R$ is said to be bounded above if there exists $M \in R$ such that $s \leq M$ for all $s \in S$. Show that if S is nonempty and bounded above, then S contains a (unique) maximum element. (That is, there exists $s_1 \in S$ such that $s \leq s_1$ for all $s \in S$.)

5. Let $n \in \mathbb{Z}^+$. Give a simple formula for $\tau(n)$, the number of positive integers which divide n, in terms of the prime factorization of n.

6. Prove that $\sqrt{5}$ and $\log_3(7)$ are not rational numbers.

7. A point $(x_0, y_0) \in \mathbb{R}^2$ is called rational if $x_0, y_0 \in \mathbb{Q}$. Prove that there are no rational points on the circle $x^2 + y^2 = 3$.

8. Prove that the arithmetic progression $\{6k + 5 : k \in \mathbb{Z}\}$ contains infinitely many primes. (Hint: You may want to follow the strategy outlined in Problem 1.4.5 in LeVeque.)