Math 109: Spring 2016
Homework 1
Due 5:00pm on Wednesday 4/6/2014

Problem 1: By using truth tables prove that, for all statements P and Q, the statement '$P \Rightarrow Q$' and its *contrapositive* '(not Q) $\Rightarrow$ (not P)' are equivalent.

What is the contrapositive of the statement 'If $f(x)$ is a differentiable function, then $f(x)$ is a continuous function.'?

Problem 2: Let P, Q, and R be statements. Write down a truth table for the statement

'[(P or Q) $\Rightarrow$ (Q and R)] and (not R)'

Problem 3: Which of the following conditions are *necessary* for the positive integer n to be divisible by 6? Which are *sufficient*? (Proofs are not necessary here.)

(1) 3 divides n.
(2) 9 divides n.
(3) 12 divides n.
(4) $n = 12$.
(5) 6 divides n^2.
(6) 2 divides n and 3 divides n.
(7) 2 divides n or 3 divides n.

Problem 4: A *tautology* is a statement which is always true. Let P and Q be statements. Which of the following four statements are tautologies? Prove that your answer is correct.

(1) '[(P $\Rightarrow$ Q) and P] $\Rightarrow$ Q.'
(2) '[(P $\Rightarrow$ Q) and Q] $\Rightarrow$ P.'
(3) '[(P $\Rightarrow$ Q) and (not P)] $\Rightarrow$ (not Q).'
(4) '[(P $\Rightarrow$ Q) and (not Q)] $\Rightarrow$ (not P).'

Problem 5: Let n be an integer. Prove that 0 divides n if and only if $n = 0$.

Problem 6: Let a, b, and c be integers. Prove that if a divides b and a divides c, then a divides $b + c$.

Problem 7: Let x and y be *non-positive* real numbers. Using only the Inequality Axioms (3.1.2 in your textbook), prove that $x < y$ implies $x^2 > y^2$.

Problem 8: Prove that the square of an even integer is even.