Math 109: Winter 2014
Homework 1
Due 5:00pm on Friday 1/10/2014

Problem 1: By using truth tables prove that, for all statements P and Q, the statement '$P \Rightarrow Q$' and its *contrapositive* '(not Q) $\Rightarrow$ (not P)' are equivalent. Write down the contrapositive of the statement 'If $f(x) = 0$, then $x > 0$.' (where f is some function from the set of real numbers to itself).

Problem 2: Use a truth table to prove that for all statements P and Q, the statement 'not(P or Q)' is equivalent to the statement '(not P) and (not Q)'. (This is one of *De Morgan's Laws*; you saw the other one in class.)

Problem 3: (Exercise 1.5 in Eccles) Which of the following conditions are *necessary* for the positive integer n to be divisible by 6? Which are *sufficient*? (Proofs are not necessary here.)

(1) 3 divides n.
(2) 9 divides n.
(3) 12 divides n.
(4) $n = 12$.
(5) 6 divides n^2.
(6) 2 divides n and 3 divides n.
(7) 2 divides n or 3 divides n.

Problem 4: Consider the following statement:

(I) "Some Californian is bad at surfing."

Which of the following are the *negation* of the statement (I)? Which of the following are *equivalent* to the statement (I)?

(1) Some Californian is not bad at surfing.
(2) Some Californians are good at surfing.
(3) All Californians are good at surfing.
(4) All Californians are not bad at surfing.
(5) All people who are not bad at surfing are Californians.
(6) All people who are bad at surfing are non-Californians.

Problem 5: Prove that 33 is an odd integer.

Problem 6: Let a, b, and c be integers. Prove that if a divides b and b divides c, then a divides c.