Math 109: Winter 2014
Homework 2
Due 5:00pm on Friday 1/17/2014

Problem 1: Prove that for all real numbers a, b, c, d we have

$$-2ab - 2ac - 2ad - 2bc - 2bd - 2cd \leq a^2 + b^2 + c^2 + d^2.$$

Problem 2: Let a be an integer. Prove that 0 divides a if and only if $a = 0$. (Hint: This problem has two parts!)

Problem 3: Let a, b, and c be integers. Prove that if a divides b or a divides c, then a divides bc.

Problem 4: For this problem, we define an integer n to be 'odd' if there is another integer q such that $n = 2q + 1$. We define an integer n to be 'even' if n is not odd.

Prove that if n is an integer and n^2 is even, then n is even.

Problem 5: What is wrong with the following "proof" that 1 is the largest integer? "Let n be the largest integer. Then, since 1 is an integer we must have $1 \leq n$. On the other hand, since n^2 is also an integer we must have $n^2 \leq n$ from which it follows that $n \leq 1$ (since n is positive). Thus, since $1 \leq n$ and $n \leq 1$ we must have that $n = 1$. Thus 1 is the largest integer as claimed."

What does this argument prove?

Problem 6: Recall from class (or from the textbook) that we have the following axioms concerning inequality of real numbers.

(1) (Trichotomy Law) For any two real number a and b, one and only one of $a < b$, $a = b$, or $a > b$ holds.
(2) (Addition Law) For any three real numbers a, b, and c, we have that $a < b$ if and only if $a + c < b + c$.
(3) (Multiplication Law) For any three real numbers a, b, and c, we have that ($a < b$ if and only if $ac < bc$, if $c > 0$) and ($a < b$ if and only if $ac > bc$, if $c < 0$).
(4) (Transitive Law) For any three real number a, b, and c, if $a < b$ and $b < c$, then $a < c$.

Use these axioms to prove by contradiction that there is no smallest positive real number.