Problem 1: Let $G$ be a simple Eulerian graph. Prove that the line graph $L(G)$ of $G$ is Eulerian. If $G$ is a simple graph such that $L(G)$ is Eulerian, is $G$ necessarily Eulerian?

Problem 2: Let $G$ be a simple graph such that $L(G)$ is Eulerian. Is $G$ necessarily Eulerian?

Problem 3: Let $G$ be a Hamiltonian graph and let $S$ be a set of $k$ vertices in $G$. Prove that $G - S$ has at most $k$ components.

Problem 4: Let $G$ be a connected graph. What can you say about

- an edge $e$ in $G$ which appears in every spanning tree?
- an edge $e$ in $G$ which appears in no spanning tree?

Problem 5: How many spanning trees does the complete bipartite graph $K_{2,s}$ have?

Problem 6: Let $\tau(G)$ be the number of spanning trees in a connected graph $G$. Prove that, for any edge $e$ in $G$, we have $\tau(G) = \tau(G - e) + \tau(G \setminus e)$.

Problem 7: Find the number of labeled trees on $1, 2, \ldots, n$ for which 1 is an end-vertex.

Problem 8: Let $G$ be a connected simple graph with vertex set $\{v_1, \ldots, v_n\}$. Define an $n \times n$ matrix $M = (m_{i,j})$ by $m_{i,i} = \deg(v_i), m_{i,i} = -1$ if $v_i$ and $v_j$ are adjacent, and $m_{i,j} = 0$ otherwise. (The matrix $M$ is called the Laplacian of the graph $G$.) The Matrix-Tree Theorem states that the number of spanning trees of $G$ is equal to any cofactor of $M$. Use the Matrix-Tree Theorem to prove that the number of spanning trees of $K_n$ is $n^{n-2}$. 
