Math 154: Winter 2017
Homework 5
Due 5:00pm on Wednesday 2/22/2017

Problem 1: Let G be a polyhedron (or polyhedral graph), each of whose faces is bounded by a pentagon or a hexagon. Use Euler's Formula to prove that G must have at least 12 pentagonal faces. Prove that if, in addition, G has exactly three faces meeting at each vertex, then G has exactly 12 pentagonal faces.

Problem 2: Let G be a simple plane graph with fewer than 12 faces, in which each vertex has degree at least 3. Use Euler's Formula to prove that G has a face bounded by at most 4 edges. Give an example to show that this can fail if G has 12 faces.

Problem 3: Let G be a Hamiltonian graph and let S be a set of k vertices in G. Prove that $G - S$ has at most k components.

Problem 4: The *k-cube* Q_k has 2^k vertices given by

$$V(Q_k) = \{(x_1, \ldots, x_k) : x_i = 0 \text{ or } 1\}$$

and an edge connecting two vertices $(x_1, \ldots, x_k)$ and $(x'_1, \ldots, x'_k)$ if and only if these vertices differ in exactly one coordinate. For which values of k is Q_k planar?

Problem 5: The *complete tripartite graph* $K_{r,s,t}$ consists of three sets of vertices

$$\{a_1, \ldots, a_r\} \cup \{b_1, \ldots, b_s\} \cup \{c_1, \ldots, c_t\}$$

with edges connecting $a_i b_j, a_i c_j$, and $b_i c_j$ for all i and j.

Problem 6: By placing vertices at $(1, 1^2, 1^3), (2, 2^2, 2^3), (3, 3^2, 3^3), \ldots$ prove that any simple graph can be drawn without crossings in three-dimensional space with every edge represented by a straight line.

Problem 7: Find a drawing of $K_{3,3}$ on the torus in which edges do not cross.

Problem 8: Use duality to show that there is no plane graph with five faces, each of which share an edge in common.

Problem 9: What is the chromatic number of the k-cube Q_k?