Math 154: Winter 2017
Homework 6
Due 5:00pm on Wednesday 3/1/2017

Problem 1: Let G be a simple graph with n vertices, each of which has degree d. Prove that

$$\chi(G) \geq \frac{n}{n-d}.$$

Problem 2: Let G be a simple plane graph with no triangles. Use Euler's Formula to show that G contains a vertex of degree at most 3.

Problem 3: Let G be a simple plane graph with no triangles. Prove that G is 4-colorable. (Use Problem 2.)

Problem 4: A graph G is called *k-critical* if $\chi(G) = k$ and if the deletion of any vertex yields a graph with smaller chromatic number. Find all 2-critical and 3-critical graphs.

Problem 5: Let G be the graph with vertex set given by the states in the United States and having an edge between two states if and only if those states share a boarder. Compute $\chi(G)$.

Problem 6: Prove that the chromatic polynomial of $K_{2,s}$ is

$$P_{K_{2,s}}(k) = k(k-1)^s + k(k-1)(k-2)^s.$$

Problem 7: Prove that the chromatic polynomial of C_n is

$$P_{C_n}(k) = (k-1)^n + (-1)^n(k-1).$$

Problem 8: Let G be a disconnected simple graph. Prove that the chromatic polynomial $P_G(k)$ of G is the product of the chromatic polynomials of the components of G. What can you say about the degree of the lowest non-vanishing term of $P_G(k)$?