Math 154: Winter 2017
Midterm 2

Instructions: This is a 50 minute exam. No books, notes, electronic devices, or interpersonal assistance are allowed. Write your answers in your blue book, making it clear what problem you are working on. Be sure to justify your answers. This exam is out of 100 points; you get 5 points for legibly writing your name on your blue book. Good luck!

Problem 1: [15 pts.] Draw the complete graph $K_5$ on the torus (i.e. genus 1 orientable surface) in such a way that the edges do not cross.

Problem 2: [20 pts.] Let $n > 2$ be an integer. How many labeled trees on the vertex set $\{1, 2, \ldots, n\}$ have both 1 and $n$ as end-vertices (i.e., leaves)?

Problem 3: [5 + 10 pts.] Let $G$ be a graph. (a) Define what it means for $G$ to be ‘planar’. (b) Give an example of a non-planar graph $G$ which has no subgraph isomorphic to $K_5$ or $K_{3,3}$.

Problem 4: [15 pts.] Let $G$ be a Hamiltonian graph and let $S$ be a set of $k$ vertices in $G$. Prove that $G - S$ has at most $k$ components.

Problem 5: [15 pts.] Find a planar graph $G$ with chromatic number $\chi(G) = 4$ such that no subgraph of $G$ is isomorphic to the complete graph $K_4$.

Problem 6: [15 pts.] Give an example of two simple graphs $G$ and $H$ which have the same number of vertices and edges but different chromatic polynomials.