Math 154: Winter 2017
Practice Midterm 1

Instructions: This is a 50 minute exam. No books, notes, electronic devices, or interpersonal assistance are allowed. Write your answers in your blue book, making it clear what problem you are working on. Be sure to justify your answers. This exam is out of 100 points; you get 5 points for legibly writing your name on your blue book. Good luck!

Problem 1: Let G be a graph. (a) Carefully define what it means for a set of edges $C^* \subseteq G$ to be a 'cutset'. (b) Prove or disprove: If G is a graph, any two cutsets in G have the same size.

Problem 2: Let n be a positive integer. Find a closed form for the expression

$$\binom{n}{0} + \binom{n}{2} + \binom{n}{4} + \cdots + \binom{n}{2\lfloor n/2 \rfloor},$$

where $\lfloor n/2 \rfloor$ is the largest integer $\leq n/2$. (Here 'closed form' means an expression without a summation sign or a $+\cdots+$.)

Problem 3: Let n be an odd positive integer and let $p_1 p_2 \ldots p_n$ be a permutation of $1, 2, \ldots, n$. Prove that the product

$$(p_1 - 1)(p_2 - 2) \cdots (p_n - n)$$

is an even integer.

Problem 4: Prove or disprove: Let G be a simple graph. The number of distinct cycles in G is no greater than the number of vertices in G.

Problem 5: [20 pts.] Let n be a positive integer. Prove that the number a_n of subsets of $\{1, 2, \ldots, n\}$ which do not contain any consecutive integers is given by the recursion

$$a_n = a_{n-1} + a_{n-2} \qquad (n \geq 3)$$

Problem 6: Let n be a positive integer. Prove the following identity:

$$\sum_{k=0}^{n} k(k-1)(k-2)\binom{n}{k} = n(n-1)(n-2)2^{n-3}.$$