Math 184A: Fall 2013
Homework 1
Due 1:00pm on Friday 10/4/2013

Problem 1: (Exercise 1.26 in Bóna) We are given 17 points inside a regular triangle of side length one. Prove that there are two points among them whose distance is not more than 1/4.

Problem 2: (Exercise 1.30 in Bóna) We select $n+1$ different integers from the set $\{1, 2, \ldots, 2n\}$. Prove that two of these selected integers will have greatest common divisor equal to 1.

Problem 3: (Exercise 1.33 in Bóna) Let r be any irrational real number. Prove that there exists a positive integer n such that the distance between nr and the closest integer is less than 10^{-10}.

Problem 4: We define an infinite sequence $a_1, a_2, \ldots$ by $a_1 = 2011, a_2 = 20112011, a_3 = 201120112011$, and so on. Prove that there are infinitely many values of n such that a_n is divisible by 2013.

Problem 5: For a positive integer n, a *permutation* p of size n is a rearrangement $p_1 p_2 \ldots p_n$ of the numbers $1, 2, \ldots, n$. For example, the permutations of size 3 are $123, 213, 132, 231, 312$, and 321. If n is an **odd** positive integer and $p = p_1 p_2 \ldots p_n$ is a permutation of size n, prove that the product

$$(1 - p_1)(2 - p_2) \cdots (n - p_n)$$

is an even integer.

Problem 6: (Exercise 2.24 in Bóna) Find (and prove!) a closed form expression for $\sum_{i=1}^{n} i(i+1)$. (Here "closed form" means no summation sign Σ and no "$+ \cdots +$".)

Problem 7: (Exercise 2.31 in Bóna) Prove that a positive integer is divisible by 3 if and only if the sum of its digits is divisible by 3.

Problem 8: (Exercise 2.34 in Bóna) Let T be a triangle. Prove that for any positive integer n, it is possible to partition T into $3n+1$ similar triangles. (Here "partition" means that we're breaking up T into smaller triangles which cover T and do not overlap.)

Problem 9: (Exercise 2.35 in Bóna) Let $n > 14$ be an integer and let S be a square. Prove that S can be partitioned into n smaller squares.