Math 184A: Fall 2013
Homework 2
Due 5:00pm on Friday 10/11/2013

Problem 1: (Exercise 3.30 in Bóna) How many four-digit positive integers are there in which all digits are different?

Problem 2: (Exercise 3.37 in Bóna) In a certain year, the month of January has five Sundays. A student wants to work five days in January, but on no more than one Sunday. In how many ways can he select five working days?

Problem 3: (Exercise 3.38 in Bóna) A host invites n couples to a party. She wants to ask a subset of the $2n$ guests to give a speech, but she does not want to ask *both* members of any couple to give speeches. In how many ways can she proceed?

Problem 4: (Exercise 3.40 in Bóna) We want to select an ordered pair (A, B) of subsets of $[n]$ so that $A \cap B \neq \emptyset$. In how many different ways can we do this?

Problem 5: (Exercise 3.48 in Bóna) In how many ways can we place n non-attacking rooks on an $n \times n$ chess board?

Problem 6: (Exercise 3.49 in Bóna) A class is attended by n sophomores, n juniors, and n seniors. In how many ways can these students form n groups of three people each if each group is to contain a sophomore, a junior, and a senior?

Problem 7: (Exercise 4.33 in Bóna) Prove, by a combinatorial argument, that for all positive integers n the number $\binom{3n}{n,n,n}$ is divisible by 6.

Problem 8: (Exercise 4.36 in Bóna) How many northeastern lattice paths are there from $(0, 0)$ to $(10, 10)$ that touch the point $(5, 5)$ but do not touch the point $(3, 3)$?

Problem 9: (Exercise 4.38 in Bóna) For all positive integers n, prove that

$$\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2.$$