**Math 184A: Fall 2013**
**Homework 4**
**Due 5:00pm on Friday 11/1/2013**

**Problem 1:** (Exercise 5.30 in Bóna) Prove that $p(n)$ grows faster than any polynomial function of $n$. That is, prove that if $f$ is any polynomial function in $n$, then there exists an integer $N$ such that $f(n) < p(n)$ for all $n > N$. (You may not use Formula 5.4 in your textbook.)

**Problem 2:** (Exercise 5.31 in Bóna) Prove that for all positive integers $n$, the inequality $p(n)^2 < p(n^2 + 2n)$ holds.

**Problem 3:** (Exercise 5.29 in Bóna) Recall that $p_k(n)$ is the number of partitions of the integer $n$ into exactly $k$ parts.
(a) Prove for all positive integers $k \leq n$, the inequality $p_k(n) \leq (n - k + 1)^{k-1}$ holds.
(b) Is it true that $p_k(n)$ is a polynomial function of $n$?

**Problem 4:** (Exercise 6.31 in Bóna) What is the number of $(2n)$-permutations whose longest cycle is of length $n$?

**Problem 5:** (Exercise 6.32 in Bóna) Let $p = p_1p_2 \ldots p_n$ be a permutation. An *inversion* of $p$ is a pair of entries $(p_i, p_j)$ such that $i < j$ but $p_i > p_j$. Let us call a permutation *even* if it has an even number of inversions and *odd* if it has an odd number of inversions. Prove that the permutation consisting of one cycle $(a_1a_2 \ldots a_k)$ is even if $k$ is odd and odd if $k$ is even.

**Problem 6:** (Exercise 6.33 in Bóna) Find a combinatorial proof of the fact that there are $\frac{n!}{2}$ even $n$-permutations.

**Problem 7:** Let $p$ be an $n$-permutation and let $A_p$ be the *permutation matrix* of $p$ (that is, $A_p$ is the $n \times n$ matrix with a 1 in position $(i, p_i)$ for $1 \leq i \leq n$ and zeroes elsewhere). Prove that $p$ is even if $\det(A_p) = 1$ and $p$ is odd if $\det(A_p) = -1$.

**Problem 8:** (Exercise 6.38 in Bóna) How many permutations $p \in S_6$ satisfy $p^3 = 1$?

**Problem 9:** (Exercise 6.39 in Bóna) How many permutations $p \in S_6$ satisfy $p^2 = 1$?