Math 184A: Fall 2013
Homework 7
Due 5:00pm on Friday 12/6/2013

Problem 1: A baker has n cupcakes in a line. He breaks the line at two places, making three segments such that the first segment can have any number of cupcakes, the second segment has an even number of cupcakes, and the third segment has an odd number of cupcakes. The baker puts red, white, or blue frosting on every cupcake in the first segment. Then he puts sprinkles or chocolate chips on every cupcake in the second section. Then he puts arsenic or cyanide in every cupcake in the third section. Let a_n be the number of ways this can be done. Find a closed form for the ordinary generating function $A(x) := \sum_{n\geq 0} a_n x^n$.

Problem 2: (Exercise 8.49 a in Bóna) Let $a_1, a_2, \ldots, a_k$ be non-negative integers and let $a(n)$ be the number of compositions of n into k parts such that the ith part is no larger than a_i Find the ordinary generating function $A(x) = \sum_{n\geq 0} a(n)x^n$.

Problem 3: There are n calculus students standing in a line. The professor breaks this line at several places into non-empty lines. She then chooses one student from each of the smaller lines to be a leader and another student from each of the smaller lines to be a scribe. Let b_n be the number of ways this can happen. Find a closed form for the ordinary generating function $B(x) = \sum_{n\geq 0} b_n x^n$.

Problem 4: There are n convicts standing in a line. The warden breaks this line at several places into non-empty lines. The warden chooses a single person from each of the smaller lines to serve time in solitary confinement. The warden then assigns every smaller line a color of T-shirt (gray, orange, or beige). Let w_n be the number of ways this can happen. Find the ordinary generating function $W(x) = \sum_{n\geq 0} w_n x^n$.

Problem 5: (Exercise 8.28 in Bóna) Let $a_0 = a_1 = 1$, and let $a_n = na_{n-1} + n(n-1)a_{n-2}$ for $n \geq 2$. Find the exponential generating function for the numbers a_n.

Problem 6: (Exercise 8.32 in Bóna) Find a closed form (no summation signs) for the generating function $G(x) = \sum_{n\geq 0} c(n,k)\frac{x^n}{n!}$.

Problem 7: (Exercise 8.43 in Bóna) We divide a set of people into subsets A, B, and C, and aske ash subset to form a line. We also require that A have an odd number of people, and that B have an even number of people. How many ways are there to do this?

Problem 8: (Exercise 8.45 in Bóna) We have n cards. We want to split them into an even number of non-empty subsets, form a line within each subset, then arrange the subsets in a line. In how many ways can we do this?

Problem 9: (Exercise 8.40 in Bóna) Let $D(n)$ be the number of derangements in S_n. Find the exponential generating function $\sum_{n\geq 0} D(n)\frac{x^n}{n!}$.