Math 184A: Fall 2013
Midterm 2
Wednesday, 11/13/2013

Instructions: Please write your name on your blue book. Make it clear in your blue book what problem you are working on. Write legibly and explain your reasoning. This exam is graded out of 100 points. Following these instructions is worth 5 points.

Problem 1: [5 + 10 points] Let n and k be positive integers. (a) Carefully define the "signless Stirling number of the first kind" $c(n, k)$ and (b) compute $c(4, 2)$.

Problem 2: [15 points] Let $g : S_9 \longrightarrow S_9$ denote the Foata bijection. Let $p = 645213897 \in S_9$ (written in one-line notation). Express the image $g(p)$ of p under g in canonical cycle form.

Problem 3: [15 points] Let n be a positive integer. Prove that the number of self-conjugate partitions of n equals the number of partitions of n with distinct odd parts.

Problem 4: [15 points] A *descent* in a permutation $p = p_1 p_2 \ldots p_n \in S_n$ (written in one-line notation) is a number $1 \leq i \leq n-1$ such that $p_i > p_{i+1}$. For example, the permutation $2\mathbf{43}\mathbf{1}\mathbf{6}5 \in S_6$ has three descents (shown in bold). Prove that if $0 \leq k < n$, the number of permutations in S_n with k descents equals the number of permutations in S_n with $n - k - 1$ descents.

Problem 5: [15 points] How many permutations $p \in S_6$ satisfy $p^2 = 1$?

Problem 6: [5 + 15 points] (a) Let n be a positive integer. Define the "partition number" $p(n)$.
(b) Prove that there exists $N > 0$ such that $p(n) > n^3$ for all $n > N$.