Math 184A: Fall 2017
Homework 3
Due 5:00pm on Wednesday 10/18/2017

Problem 1: (Exercise 4.33 in Bóna) Let n be a positive integer. Use a combinatorial argument to show that $\binom{3n}{n,n,n}$ is divisible by 6.

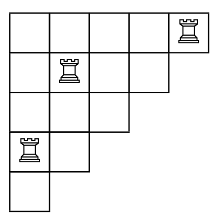
Problem 2: (Exercise 4.45 in Bóna) Let n be a positive integer. Prove the following formula:

$$\sum_{\substack{k=0 \\ k \text{ even}}}^{n} \binom{n}{k} 2^k = \frac{3^n + (-1)^n}{2}.$$

Problem 3: (Exercise 5.28 in Bóna) Find a closed formula for $S(n, n-3)$, for all $n \geq 3$.

Problem 4: Let $n = 2k$ be an even positive integer. How many (strong) compositions of n are there into even parts?

Problem 5: Let n be a positive integer and let B_n be the 'triangular' chessboard with left-justified rows consisting of $n-1, n-2, \ldots, 1$ squares from top to bottom. The board B_6 is shown below, with three non-attacking rooks. Prove that the number of ways to place r non-attacking rooks on B_n equals the number of set partitions of $[n]$ with $n - r$ blocks.



Problem 6: (Exercise 5.34 in Bóna) Let $B_k(n)$ be the number of set partitions of $[n]$ such that if i and j are in the same block, then $|i-j| > k$. Prove that $B_k(n) = B(n-k)$ for all $n \geq k$. (Hint: Think about the previous problem.)

Problem 7: (Exercise 5.30 in Bóna) Prove that the partition number $p(n)$ grows faster than any polynomial function of n. You may not use Formula 5.4 in your textbook. (Hint: It is enough to show that, for a fixed positive integer k, we have $p(n) > n^k$ for n sufficiently large.)

Problem 8: Let $F(n)$ be the number of set partitions of $[n]$ with no singleton blocks. Give a bijective proof of the identity $B(n) = F(n) + F(n+1)$.

Problem 9: (Exercise 5.35 in Bóna) Let a_n be the number of (strong) compositions of n into parts which are larger than 1. Express a_n in terms of a_{n-1} and a_{n-2}.