Math 184A: Fall 2017
Midterm 2
Friday, 11/17/2017

Instructions: Please write your name on your blue book. Make it clear in your blue book what problem you are working on. Write legibly and justify your answers. No books, notes, or electronic devices are allowed. This exam is graded out of 100 points. Following these instructions is worth 5 points.

Problem 1: [15 points] Let $n$ be a positive integer. Give an explicit formula for the signless Stirling number of the first kind $c(n, 1)$.

Problem 2: [15 points] How many permutations $\pi \in S_6$ satisfy $\pi^3 = 1$?

Problem 3: [20 points] For any positive integer $n$, let $C(n)$ be the number of set partitions of $[n]$ which do not have any singleton blocks. Find a formula for $C(n)$ in terms of the Bell numbers $B(n)$.

Problem 4: [5 + 20 points] (a) Let $(a_n)_{n \geq 0}$ be a sequence of real numbers. Define the “ordinary generating function” $A(x)$ of $(a_n)$.

(b) Find a closed form for $A(x)$ if $a_n$ is defined recursively by $a_0 = 1$, $a_1 = 3$, and $a_n = 3a_{n-1} + 2a_{n-2}$ for all $n \geq 2$.

Problem 5: [25 points] A baker has $n$ cupcakes in a line, but only 3 cherries. The baker splits the line in two places to make three smaller lines, and then places a cherry on one cupcake in each of these smaller lines. Let $c_n$ be the number of ways the baker can do this. Find a closed form for the ordinary generating function $C(x)$ of the sequence $(c_n)_{n \geq 0}$. 