**Math 190: Fall 2014**
**Homework 8**
**Due 5:00pm on Friday 12/12/2014**

**Problem 1:** (Exercise 30.4 in Munkres) Prove that every compact metrizable space $X$ has a countable basis (i.e., is second countable). (Hint: Consider finite coverings $\mathcal{A}_n$ of $X$ by balls of radius $1/n$.)

**Problem 2:** Let $X$ be a metrizable space which has a countable dense subset (i.e., is separable). Show that $X$ has a countable basis.

**Problem 3:** Let $f : X \to Y$ be a continuous surjection. If $X$ is Lindelöf, prove that $Y$ is Lindelöf.

**Problem 4:** (Exercise 31.1 in Munkres) If $X$ is regular, show that every pair of points in $X$ have neighborhoods whose closures are disjoint.

**Problem 5:** (Exercise 31.2 in Munkres) If $X$ is normal, show that every pair of disjoint closed sets in $X$ have neighborhoods whose closures are disjoint.

**Problem 6:** (Exercise 31.3 in Munkres) Show that every order topology is regular.

**Problem 7:** (Exercise 32.1 in Munkres) Show that a closed subspace of a normal space is normal.

**Problem 8:** (Exercise 32.3 in Munkres) Show that every locally compact Hausdorff space $X$ is regular. (Hint: Imbed $X$ in its one-point compactification, which is compact Hausdorff and hence regular.)

**Problem 9:** Let $M$ denote the Möbius band and let $D$ denote the disk $\{(x, y) : x^2 + y^2 \leq 1\}$ in $\mathbb{R}^2$. Let $\sim$ denote the equivalence relation on $M \amalg D$ obtained by identifying a point on the boundary circle of $M$ with the corresponding point on the boundary circle of $D$. Prove that the quotient space $(M \amalg D)/\sim$ is homeomorphic to 2-dimensional real projective space $P^2$. (Hint: Think of $P^2$ as the sphere $S^2$ with antipodal points identified. Consider the 'arctic circle' $C = S^2 \cap \{z = 1/2\}$ and the corresponding 'antarctic circle' $C' = S^2 \cap \{z = -1/2\}$. Consider equivalence classes of points in the polar and tropical regions.)