Math 190: Fall 2014
Midterm 1

Instructions: This is a 50 minute exam. No books, notes, electronic devices, or interpersonal assistance are allowed. Write your answers in your blue book, making it clear what problem you are working on. Be sure to justify your answers. This exam is out of 115 points; you get 5 points for legibly writing your name on your blue book. Good luck!

Problem 1: [5 + 15 points] (a) Carefully define the "lower limit topology" $\mathbb{R}_\ell$ on the set of real numbers. (b) To what point or points (if any) does the sequence $x_n = 3 + \frac{(-1)^n}{n}$ converge to in $\mathbb{R}_\ell$?

Problem 2: [20 points] Let X be a set and let $\{\mathfrak{T}_\alpha\}_{\alpha \in I}$ be a nonempty family of topologies on X. Prove or disprove: the union $\bigcup_{\alpha \in I} \mathfrak{T}_\alpha$ is necessarily a topology on X.

Problem 3: [15 points] Endow the set $\mathbb{R} \times \mathbb{Z}$ with the dictionary order $<$. That is, for $(x, r), (x', r') \in \mathbb{R} \times \mathbb{Z}$, we have $(x, r) < (x', r')$ if $x < x'$ (as real numbers) or $x = x'$ and $r < r'$ (as integers). Prove that the order topology on $\mathbb{R} \times \mathbb{Z}$ is the discrete topology.

Problem 4: [5 + 15 points] Let X be a topological space. (a) Carefully define what it means for X to be "Hausdorff". (b) Give an example of a topological space X which is not Hausdorff but which has the "T_1 property" that $\{x\}$ is a closed set for all $x \in X$.

Problem 5: [5 + 15 points] Let X and Y be topological spaces and let $y_0 \in Y$ be a point. (a) Give a basis for the "product topology" on $X \times Y$. (b) Prove that the function $f : X \to X \times Y$ defined by $f(x) = (x, y_0)$ is continuous (where $X \times Y$ is endowed with the product topology).

Problem 6: [15 points] Consider the real numbers $\mathbb{R}$ with their standard topology and endow the closed interval $[-1, 2] \subset \mathbb{R}$ with the subspace topology. Consider the following subsets of $[-1, 2]$: $A = (0, 1), B = (1, 2], C = \{-1\} \cup (1, 2]$. Which are open in $\mathbb{R}$? Which are open in $[-1, 2]$?