Math 190: Fall 2014
Final Exam

Instructions: This is a 3 hour exam. No books, notes, electronic devices, or inter-personal assistance are allowed. Write your answers in your blue book, making it clear what problem you are working on. Be sure to justify your answers. This exam is out of 140 points; you get 5 points for legibly writing your name on your blue book. Good luck!

Problem 1: [5+5] (a) Carefully define what it means for a space X to be "path connected". (b) Give an example of a connected space which is not path connected. You need not prove that your example works.

Problem 2: [5+10] (a) Carefully define what it means for a space X to be "locally compact". (b) Prove that $\mathbb{Q}$ is not locally compact.

Problem 3: [10] Is $\mathbb{R}_\ell$ connected? Justify your claim.

Problem 4: [15] Let X be the "θ-space":

$$X = \{(x, y) \in \mathbb{R}^2 \,:\, x^2 + y^2 = 1\} \cup \{(x, 0) \in \mathbb{R}^2 \,:\, -1 \leq x \leq 1\}.$$

Prove that X is not homeomorphic to the circle S^1.

Problem 5: [10] Give an example of metric spaces (X, d_X) and (Y, d_Y) such that

- X and Y are homeomorphic,
- X is bounded, and
- Y is not bounded.

Problem 6: [10] Let $I = [0, 1]$. Prove that there is no continuous bijection $f : I \to I^2$.

Problem 7: [10] Give an example of a continuous bijection $f : X \to Y$ of topological spaces which is not a homeomorphism.

Problem 8: [15] Prove that $[0, 1]^\omega = [0, 1] \times [0, 1] \times \cdots$ is not compact in the box topology. (The Tychonoff Theorem implies that this space *is* compact in the *product* topology.)

Problem 9: [5+10] Let X be a topological space and let $\sim$ be an equivalence relation on X. (a) Carefully define the "quotient topology" on the set $X/\sim$ (i.e., what are the open sets?). (b) Give an example of a space X and an equivalence relation $\sim$ such that X is Hausdorff but $X/\sim$ is not. You need not prove that your example works.

Problem 10: [5+20] Define an equivalence relation $\sim$ on $\mathbb{R}$ by $x \sim y$ if and only if $x - y \in \mathbb{Z}$. (a) Describe the *points* in the quotient space $\mathbb{R}/\sim$ (these are *sets of points* in $\mathbb{R}$). (b) Prove that $\mathbb{R}/\sim$ is homeomorphic to the circle S^1. (To get full credit, you need to give an explicit and justified homeomorphism.)