Math 190: Winter 2016
Homework 7
Due 5:00pm on Wednesday 3/2/2016

Problem 1: Let \( X \) be a compact Hausdorff space. Prove that \( X \) is regular. That is, one-point sets are closed in \( X \) and

given \( x \in X \) and a closed set \( A \subset X \) such that \( x \notin A \), we have disjoint open sets \( U \) and \( V \) such that \( x \in U \) and \( A \subset V \).

Problem 2: Let \( X \) be the ‘\( \theta \)-space’:

\[
X = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\} \cup \{(x,0) : -1 \leq x \leq 1\}.
\]

Prove that \( X \) is not homeomorphic to \( S^1 \).

Problem 3: Suppose \( X \) is locally path connected. Show that every open connected set in \( X \) is path connected.

Problem 4: Let \( X \) be an order topology in which every closed interval is compact. Show that \( X \) has the least upper bound property. (That is, every nonempty subset \( S \subseteq X \) which is bounded above has a least upper bound.)

Problem 5: Prove that \([0,1]\) is not limit point compact as a subspace of \( \mathbb{R}_\ell \).

Problem 6: Let \( f : X \to Y \) be a continuous function. If \( X \) is locally compact, is \( f(X) \) necessarily locally compact?

Problem 7: Let \( P^n \) denote \( n \)-dimensional real projective space. Prove that \( P^n \) is homeomorphic to \( D^n/\sim \), where we declare \( x \sim -x \) whenever \( x \in S^{n-1} \subset D^n \).

Problem 8: Let \( M \) be the Möbius strip and let \( D = D^2 \) be the 2-dimensional disc. Prove that the quotient space \( X = (M \amalg D)/\sim \) obtained by gluing the boundary circle of \( M \) to the boundary circle of \( D \) is homeomorphic to 2-dimensional projective space \( P^2 \). (Constructing the actual maps is tedious here. Intuitive ‘cut and paste’ arguments are acceptable. Hint: Consider the ‘arctic and antarctic’ circles on \( S^2 \).)