Math 190: Winter 2016
Homework 7
Due 5:00pm on Wednesday 3/2/2016

Problem 1: Let X be a compact Hausdorff space. Prove that X is *regular*. That is, one-point sets are closed in X and

> given $x \in X$ and a closed set $A \subset X$ such that $x \notin A$, we have disjoint open sets U and V such that $x \in U$ and $A \subset V$.

Problem 2: Let X be the 'θ-space':

$$X = \{(x, y) \in \mathbb{R}^2 \,:\, x^2 + y^2 = 1\} \cup \{(x, 0) \,:\, -1 \leq x \leq 1\}.$$

Prove that X is *not* homeomorphic to S^1.

Problem 3: Suppose X is locally path connected. Show that every open connected set in X is path connected.

Problem 4: Let X be an order topology in which every closed interval is compact. Show that X has the least upper bound property. (That is, every nonempty subset $S \subseteq X$ which is bounded above has a least upper bound.)

Problem 5: Prove that $[0, 1]$ is not limit point compact as a subspace of $\mathbb{R}_\ell$.

Problem 6: Let $f : X \to Y$ be a continuous function. If X is locally compact, is $f(X)$ necessarily locally compact?

Problem 7: Let P^n denote n-dimensional real projective space. Prove that P^n is homeomorphic to $D^n/\sim$, where we declare $\mathbf{x} \sim -\mathbf{x}$ whenever $\mathbf{x} \in S^{n-1} \subset D^n$.

Problem 8: Let M be the Möbius strip and let $D = D^2$ be the 2-dimensional disc. Prove that the quotient space $X = (M \amalg D)/\sim$ obtained by gluing the boundary circle of M to the boundary circle of D is homeomorphic to 2-dimensional projective space P^2. (Constructing the actual maps is tedious here. Intuitive 'cut and paste' arguments are acceptable. Hint: Consider the 'arctic and antarctic' circles on S^2.)