Math 190: Winter 2016
Midterm 2

Instructions: This is a 50 minute exam. No books, notes, electronic devices, or interpersonal assistance are allowed. Write your answers in your blue book, making it clear what problem you are working on. Be sure to justify your answers. This exam is out of 100 points; you get 5 points for legibly writing your name on your blue book. Good luck!

Problem 1: [10 pts.] Let $\mathbb{R}_\ell$ be the real line in the lower limit topology. Is $\mathbb{R}_\ell$ connected?

Problem 2: [15 pts.] Let X be an infinite set with the finite complement topology. Prove that X is not metrizable.

Problem 3: [15 pts.] Let $\mathbb{R}$ be the real line with its standard topology and let $\mathbb{R}/\mathbb{Q}$ be the quotient space obtained by identifying $\mathbb{Q} \subset \mathbb{R}$ to a point. Prove that within $\mathbb{R}/\mathbb{Q}$ we have $\overline{\{\mathbb{Q}\}} = \mathbb{R}/\mathbb{Q}$.

Problem 4: [15 pts.] Prove that the half-open interval $[0, 1)$ and the open interval $(0, 1)$ are not homeomorphic.

Problem 5: [5 + 20 pts.] Define an equivalence relation $\sim$ on $\mathbb{R}^2$ by $(x_1, y_1) \sim (x_2, y_2)$ if and only if $y_1 - x_1^2 = y_2 - x_2^2$. Give the set $\mathbb{R}^2/\sim$ the quotient topology.

(a) Describe the *points* of $\mathbb{R}^2/\sim$. (Hint: These are *sets of points* in $\mathbb{R}^2$.)

(b) Prove that $\mathbb{R}^2/\sim$ is homeomorphic to $\mathbb{R}$.

Problem 6: [15 pts.] Give $\mathbb{R}^\omega = \mathbb{R} \times \mathbb{R} \times \cdots$ the *box* topology. Consider the sequence $\mathbf{x}_1, \mathbf{x}_2, \cdots \in \mathbb{R}^\omega$ given by

$$\mathbf{x}_n = (1/n, 1/n, 1/n, \dots).$$

Let $\mathbf{0} = (0, 0, 0, \dots) \in \mathbb{R}^\omega$. Do we have that $\mathbf{x}_n \to \mathbf{0}$ as $n \to \infty$?