Math 190: Winter 2018
Midterm 1

Instructions: This is a 50 minute exam. No books, notes, electronic devices, or interpersonal assistance are allowed. Write your answers in your blue book, making it clear what problem you are working on. Be sure to justify your answers. This exam is out of 100 points; you get 5 points for legibly writing your name on your blue book. Good luck!

Problem 1: [20 pts.] Let $S^1 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ be the circle, let $I = [0, 1]$ be the interval, and let A be the annulus

$$A := \{(x, y) \in \mathbb{R}^2 : 1 \leq x^2 + y^2 \leq 4\}.$$

Prove that $S^1 \times I$ is homeomorphic to A.

Problem 2: [5 + 15 pts.] Let X be a topological space.
(a) Carefully state what it means for X to be "Hausdorff".
(b) Prove that X is Hausdorff if and only if the diagonal $\Delta = \{(x, x) : x \in X\}$ is closed in $X \times X$.

Problem 3: [5 + 15 pts.] Consider the set $\mathbb{R}^\omega = \mathbb{R} \times \mathbb{R} \times \cdots$ of all real sequences.
(a) Carefully define the "box topology" on $\mathbb{R}^\omega$.
(b) Is the function $f : \mathbb{R} \to \mathbb{R}^\omega$ given by $f(t) = (t, t, t, \dots)$ continuous if $\mathbb{R}^\omega$ has the box topology?

Problem 4: [15 pts.] Recall that the *(real) special linear group* is given by

$$SL_n(\mathbb{R}) = \{A \in \mathrm{Mat}_n(\mathbb{R}) : \det(A) = 1\}.$$

Prove that $SL_n(\mathbb{R})$ is closed in matrix space $\mathrm{Mat}_n(\mathbb{R})$.

Problem 5: [20 pts.] Endow the square $I^2 = [0, 1] \times [0, 1]$ with the dictionary order, and consider the corresponding order topology $\mathcal{T}$. Is the set $\mathcal{B}$ of 'rational dictionary order intervals' of the form

$$\begin{aligned} &[0 \times 0, c \times d), \\ &(a \times b, 1 \times 1], \text{ or} \\ &(a \times b, c \times d) \end{aligned}$$

where $a, b, c, d \in \mathbb{Q}$ are **rational** numbers a basis for the topology $\mathcal{T}$?