Math 190: Winter 2018
Midterm 2

Instructions: This is a 50 minute exam. No books, notes, electronic devices, or interpersonal assistance are allowed. Write your answers in your blue book, making it clear what problem you are working on. Be sure to justify your answers. This exam is out of 100 points; you get 5 points for legibly writing your name on your blue book. Good luck!

Problem 1: [15 pts.] Let X be an infinite set. Prove that X is connected in the finite complement topology.

Problem 2: [15 pts.] Give an example of a Hausdorff space X and an equivalence relation $\sim$ on X such that the quotient space $X/\sim$ is not Hausdorff. Why does your example work?

Problem 3: [5 + 15 pts.] Let X be a topological space.
(a) Define what it means for X to be 'locally compact'.
(b) If $f : X \to Y$ is a continuous surjection between spaces and X is locally compact, is Y necessarily locally compact?

Problem 4: [15 pts.] Let $T : S^2 \to \mathbb{R}$ be a continuous function defined on the sphere $S^2 = \{x \in \mathbb{R}^3 : ||x|| = 1\}$. Prove there is a point $x \in S^2$ such that $T(x) = T(-x)$.

Problem 5: [20 + 10 pts.] Consider the closed disc $D^2 = \{(x_1, x_2) \in \mathbb{R}^2 : x_1^2 + x_2^2 \leq 1\}$. Let S^1 be the boundary circle in D^2 and let $S^2 \subset \mathbb{R}^3$ be the unit sphere centered at the origin. Endow D^2/S^1 with the quotient topology.
(a) Describe a continuous bijection $f : (D^2/S^1) \longrightarrow S^2$. You need to show why your map f 'works'.
(b) Explain why the map f constructed in (a) is in fact a homeomorphism.