Problem 1: Let $\mathbb{Z}^n \subset \mathbb{C}^n$ be the integer lattice. Prove that the corresponding ideal $I(\mathbb{Z}^n) \subseteq \mathbb{C}[x_1, \ldots, x_n]$ is given by $I(\mathbb{Z}^n) = 0$.

Problem 2: Let $\mathbb{K}$ be any field and let $a = (a_1, \ldots, a_n) \in \mathbb{K}^n$ be a point in affine $n$-space. Prove that
$$I(\{a\}) = \langle x_1 - a_1, \ldots, x_n - a_n \rangle \subseteq \mathbb{K}[x_1, \ldots, x_n].$$

Problem 3: Let $R$ be a ring. The radical of an ideal $I \subseteq R$ is
$$\sqrt{I} := \{ r \in R : r^n \in I \text{ for some } n > 0 \}.$$
Prove that $\sqrt{I}$ is an ideal of $R$ containing $I$.
Prove that if $R = \mathbb{K}[x_1, \ldots, x_n]$ and $X \subseteq \mathbb{K}^n$, then $\sqrt{I(X)} = I(X)$. (Ideals $I$ with $\sqrt{I} = I$ are called radical.)

Problem 4: Let $<$ be any monomial order on $\mathbb{K}[x_1, \ldots, x_n]$ and prove the following statements for $f, g \in \mathbb{K}[x_1, \ldots, x_n]$ or give counterexamples.

1. $LT(f \cdot g) = LT(f) \cdot LT(g)$.
2. $LT(f + g) = \max(LT(f), LT(g))$.

Problem 5: Cox-Little-O’Shea Exercise 2.3.5.

Problem 6: Cox-Little-O’Shea Exercise 2.3.7.

Problem 7: Cox-Little-O’Shea Exercise 2.4.5.

Problem 8: Cox-Little-O’Shea Exercise 2.4.8.

Problem 9: Cox-Little-O’Shea Exercise 2.5.2.