Math 202C: Spring 2015
Homework 3
Due 5/4/2015

Problem 1: (Math 202 Qual, Spring 2013) Let S be the parametric surface defined by

$$\begin{cases} x = u - 2v \\ y = uv \\ z = v \end{cases}$$

(1) Compute the reduced Gröbner basis for the ideal generated by this set of equations relative to the lexicographic order where $u > v > x > y > z$.
(2) Find the equation of the smallest variety V containing S.
(3) Show that $S = V$.

Problem 2: Let $\mathbb{K}$ be an algebraically closed field and let $I \subseteq \mathbb{K}[x_1, \ldots, x_n]$ be an ideal. Prove that the quotient $\mathbb{K}[x_1, \ldots, x_n]/I$ is a finite-dimensional $\mathbb{K}$-vector space if and only if the variety $\mathbf{V}(I)$ is a finite subset of $\mathbb{K}^n$. What happens when $\mathbb{K}$ is not algebraically closed?

Problem 3: Cox-Little-O'Shea 3.3.6.

Problem 4: Cox-Little-O'Shea 4.2.15.

Problem 5: Cox-Little-O'Shea 4.3.11.

Problem 6: (Math 202 Qual, Fall 2011) Let $I = \langle x^3 y \rangle$ and $J = \langle xy^3 + xy \rangle$ be ideals in $\mathbb{K}[x, y]$. Find a Gröbner basis for $I \cap J$ relative to the lexicographic order where $x > y$.