Instructions: Write your proofs clearly and legibly in complete sentences. Unless otherwise indicated, you must show all work. You may freely use any results we learned in class.
Warning - This side will not be graded
1. Let \( A \) and \( B \) be sets of positive real numbers. Prove that

\[
\inf\{a \cdot b : a \in A, b \in B\} = (\inf A) \cdot (\inf B).
\]
Warning - This side will not be graded
2. Let \((X, d)\) be a metric space and let \(p \in X\). Prove that the set \(E = \{q \in X : d(p, q) > 1\}\) is open.
Warning - This side will not be graded
3. Let $A$ be the set of all infinite subsets of $\mathbb{N}$. Is $A$ countable or uncountable? Prove your answer.
Warning - This side will not be graded
4. Let $b, y \in \mathbb{R}$ with $b > 1$. Prove that there is $n \in \mathbb{Z}_+$ with $b^n > y$. 
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