Instructions:

- Formatting: Use your own sheets of paper. You do not need to rewrite the questions - just write the problem number.
- Allowed Resources: You may use your textbook, your lecture notes, your homework assignments, and your scratch-work from this class.
- Prohibited Resources: You are not allowed to seek help from other human beings, online resources, or any other resources not affiliated with our class. If I or your TA find anything suspicious about your solutions or your exam performance, you may be required to have a video chat with me where I may ask you about your solutions or ask you how to solve similar problems. Any evidence of cheating will be taken seriously and may have severe consequences.
- Solutions: Your solutions should be written clearly, in complete sentences, and show all justifying work. In your solutions you can apply theorems that we learned in class (unless the problem says otherwise), but you cannot apply results from exercises or homework (those materials are for reference only).
- Questions: If you have any questions about the exam, I will be standing by - just email me at bse-ward@ucsd.edu.
- Finishing: The exam ends at 1:50 pm. After finishing the exam, you must scan or take photos of your solutions and upload them to Gradescope. You are allowed 10 minutes to upload your solutions - the deadline for submission on Gradescope is 2:00 pm. If you have any technical difficulties uploading your work to Gradescope, you must email me the images or pdf of your solutions by 2:00 pm.

1. (10 points) Let \( f : (0, 2) \to \mathbb{R} \) be differentiable on \((0, 2)\). Prove that if \( f' \) is bounded then \( \lim_{n \to \infty} f\left(\frac{1}{n}\right) \) exists.

2. (10 points) Let \( n \in \mathbb{Z}_+ \) and let \( f : \mathbb{R} \to \mathbb{R} \) be a function such that \( f^{(2n)} \) is defined and continuous on \( \mathbb{R} \). Assume that \( f'(0) = f''(0) = \cdots = f^{(2n-1)}(0) = 0 \) and that \( f^{(2n)}(0) > 0 \). Prove that \( f(x) \) has a local minimum at \( x = 0 \). (Warning: it may be that \( f^{(2n+1)} \) is not defined anywhere).

3. (10 points) Let \( a, b \in \mathbb{Q} \), let \( f : [a, b] \to \mathbb{R} \) be bounded, and let \( P = \{x_0, x_1, \ldots, x_n\} \) be a partition of \([a, b]\). Set \( \Delta x_i = x_i - x_{i-1} \) and \( M_i = \sup_{x_{i-1} \leq x \leq x_i} f(x) \) so that

\[
U(P, f) = \sum_{i=1}^{n} M_i \Delta x_i.
\]

Let \( \epsilon > 0 \). Prove there is a partition \( Q \) of \([a, b]\) satisfying \( Q \subseteq Q \) and

\[
U(Q, f) < U(P, f) + \epsilon.
\]

(Hint: for every \( 0 < i < n \), replace \( x_i \) with two points \( p_i, q_i \in \mathbb{Q} \).)