Taylor's Thm: Why $\forall k \leq n$ $P^{(k)}(\alpha) = f^{(k)}(\alpha)$?

$$P(t) = \sum_{m=0}^{n-1} \frac{f^{(m)}(\alpha)}{m!}(t-\alpha)^m$$

$$P^{(k)}(t) = \sum_{m=0}^{n-1} \frac{f^{(m)}(\alpha)}{m!}\, m\cdot(m-1)\cdots(m-k+1)(t-\alpha)^{m-k}$$

$$P^{(k)}(\alpha) = \sum_{m=0}^{n-1} \underbrace{\frac{f^{(m)}(\alpha)}{m!}\cdot m(m-1)\cdots(m-k+1)\cdot 0^{m-k}}_{\text{equals}}$$

equals:

- 0 if $m \leq k-1$ because $m(m-1)\cdots(m-k+1)=0$
- 0 if $m > k$ because $0^{m-k}=0$
- $\frac{f^{(m)}(\alpha)}{m!}\cdot m(m-1)\cdots 2\cdot 1 = f^{(k)}(\alpha)$ if $m=k$ because $0^{m-k}=1$

$$= f^{(k)}(\alpha)$$