1. Prove that if $f : X \to Y$ and $g : Y \to Z$ are bijections then $g \circ f$ is a bijection.

2. Let $f : X \to Y$ and $g : Y \to Z$ be functions. Prove that if $g \circ f$ is injective then $f$ is injective.

3. Let $f : X \to Y$ and $g : Y \to X$ be functions. Prove that $g$ is the inverse of $f$ if and only if $g \circ f = \text{id}_X$ and $f \circ g = \text{id}_Y$.

4. Let $f : X \to Y$ be a function with $X \neq \emptyset$. Prove that $f$ is injective if and only if there is a function $g : Y \to X$ with $g \circ f = \text{id}_X$. (Hint: for help building $g$, review the proof that if $f$ is bijective then $f$ is invertible).

5. Let $f : X \to Y$ and $g_1, g_2 : Y \to Z$ be functions. Assume that $g_1 \neq g_2$ and that $f$ is surjective. Prove that $g_1 \circ f \neq g_2 \circ f$.

6. Let $X$ be a finite set and let $A, B \subseteq X$. Prove that if $|A| + |B| \geq |X| + 5$ then $|A \cap B| \geq 5$. (You may use the fact that if $Y \subseteq X$ then $|Y| \leq |X|$).