Math 20B
Midterm #1
October 19, 2016

Turn off and put away your cell phone.
You may not use any calculators during this exam.
You may not use any cheat sheets during this exam.
Read each question carefully, and answer each question completely.
Show all of your work; no credit will be given for unsupported answers.
Write your solutions clearly and legibly; no credit will be given for illegible solutions.
If any question is not clear, ask for clarification.

ANSWERS

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1. (25 points) Calculate the area between the curves \( f(x) = x + 6 \), \( g(x) = -x - 2 \), and \( h(x) = x^3 \).

(Hint: below is a picture to help you, but note that the picture is not drawn to scale.)

**ANSWER:**

\[
\text{Area} = \int_{a}^{b} \left( y_{\text{top}} - y_{\text{bottom}} \right) \, dx
\]

\[
= \int_{-4}^{2} \left( (x+6) - (-x-2) \right) \, dx + \int_{-1}^{2} \left( (x+6) - (x^3) \right) \, dx
\]

\[
= \left[ x^2 + 8x \right]_{-4}^{2} + \left[ -\frac{1}{4} x^4 + \frac{1}{2} x^2 + 6x \right]_{-1}^{2}
\]

\[
= \left( [(2)^2 + 8(2)] - [(-4)^2 + 8(-4)] \right) + \left( [-\frac{1}{4} (2)^4 + \frac{1}{2} (2)^2 + 6(2)] - [-\frac{1}{4} (-1)^4 + \frac{1}{2} (-1)^2 + 6(-1)] \right)
\]

\[
= \frac{99}{4}
\]
2. (25 points) Calculate the volume $V$ of the solid obtained by rotating the region under 
\[ y = x \sqrt{\sin \left( \frac{\pi}{2} x^3 \right)} \]
about the $x$-axis for $0 \leq x \leq 1$.

**ANSWER:**

Volume = \[ \pi \int_{a}^{b} r^2 \, dx = \pi \int_{0}^{1} x^2 \sin \left( \frac{\pi}{2} x^3 \right) \, dx \]

Integration by substitution:
Let \( u = \frac{\pi}{2} x^3 \), \( du = \frac{3}{2} \pi x^2 \, dx \)

then volume = \( \frac{3}{2} \pi \int_{0}^{\pi/2} \sin u \, du = \frac{2}{3}(-\cos u) \bigg|_{0}^{\pi/2} = \frac{2}{3} \)
3. (25 points) Evaluate $\int x^2 \sin(x)\,dx$.

**ANSWER:**
Integration by parts:
Let $u = x^2$, $v' = \sin x$
and $u' = 2x$, $v = -\cos x$
then $\int x^2 \sin(x)\,dx = -x^2 \cos x + 2 \int x \cos x \,dx$
Integration by parts again:
Let $u = x$, $v' = \cos x$
and $u' = 1$, $v = \sin x$
then $\int x^2 \sin(x)\,dx = -x^2 \cos x + 2 \int x \cos x \,dx = -x^2 \cos x + 2x \sin x - 2 \int \sin x \,dx$
$= -x^2 \cos x + 2x \sin x + 2 \cos x + C$
4. (25 points) Calculate \((-\sqrt{3}+i)^8\). Write your answer in standard form. (i.e. complex Cartesian form)

**ANSWER:**

\[-\sqrt{3} + i\]  \(= (2 (\cos \frac{\pi}{6} \pi + i \sin \frac{\pi}{6} \pi))^8 = (2^8 (\cos \frac{4\pi}{6} \pi + i \sin \frac{4\pi}{6} \pi)) = (256 (\frac{-1}{2} + i \frac{\sqrt{3}}{2})) = -128 + (128\sqrt{3})i\]