3.2.4. The given system is equivalent to $Ax = b$ where $A = \begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}$ and $b = \begin{bmatrix} 9 \\ -4 \end{bmatrix}$. Using Cramer’s rule, we compute:

$$A_1(b) = \begin{bmatrix} 9 & 2 \\ -4 & -1 \end{bmatrix} \quad \text{and} \quad A_2(b) = \begin{bmatrix} -5 & 9 \\ 3 & -4 \end{bmatrix},$$

with $\det(A) = -1 \neq 0$, $\det(A_1(b)) = -1$, and $\det(A_2(b)) = -7$.

Therefore, the solution is given by

$$x_1 = \frac{\det(A_1(b))}{\det(A)} = 1 \quad \text{and} \quad x_2 = \frac{\det(A_2(b))}{\det(A)} = 7.$$

3.2.6. The given system is equivalent to $Ax = b$ where $A = \begin{bmatrix} 1 & 3 & 1 \\ -1 & 0 & 2 \\ 3 & 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}$. Using Cramer’s rule, we compute:

$$A_1(b) = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 0 & 2 \\ 2 & 1 & 0 \end{bmatrix}, \quad A_2(b) = \begin{bmatrix} 1 & 4 & 1 \\ -1 & 2 & 2 \\ 3 & 2 & 0 \end{bmatrix}, \quad A_3(b) = \begin{bmatrix} 1 & 3 & 4 \\ -1 & 0 & 2 \\ 3 & 1 & 2 \end{bmatrix},$$

with $\det(A) = 15 \neq 0$, $\det(A_1(b)) = 6$, $\det(A_2(b)) = 12$ and $\det(A_3(b)) = 18$.

Therefore, the solution is given by

$$x_1 = \frac{\det(A_1(b))}{\det(A)} = \frac{2}{5}, \quad x_2 = \frac{\det(A_2(b))}{\det(A)} = \frac{4}{5}, \quad \text{and} \quad x_3 = \frac{\det(A_3(b))}{\det(A)} = \frac{6}{5}.$$

3.2.20. The parallelogram is determined by the columns of $A = \begin{bmatrix} -2 & 4 \\ 4 & -5 \end{bmatrix}$ so the area of the parallelogram is $|\det(A)| = |-6| = 6$.

3.2.22. We first translate the given coordinates of the parallelogram so that the origin becomes one of the vertices. Subtracting $(-2, 0)$ from each vertex gives a new set of vertices:

$$(0, 0), (5, 0), (-3, 3), (2, 3).$$

This new parallelogram is determined by the columns of $A = \begin{bmatrix} 5 & -3 \\ 0 & 3 \end{bmatrix}$ so the area of the parallelogram is $|\det(A)| = |15| = 15$.

3.2.24. The parallelepiped is determined by the columns of $A = \begin{bmatrix} 1 & -2 & -1 \\ 3 & 0 & 3 \\ 0 & 2 & -1 \end{bmatrix}$ so the area of the parallelepiped is $|\det(A)| = |-18| = 18$. 

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