Homework problems due Friday, April 20:

In text, Sec. 7.1, pp. 321–322: 1 (See Th. 7.1.3.), 9, 11, 14 (\(a_{11}\) must also be nonzero for the matrices in \(G\)). In addition for problem 14 prove that \(G/N \cong \mathbb{R}^x\).

Added Problems (These are also part of the homework.):

1. Let \(G\) be a \(p\)-group, i.e., \(G\) is a finite group with \(|G| = p^n\) for some prime number \(p\) and positive integer \(n\). For this problem you will need to use the fact that if \(G\) is any \(p\)-group, then its center is nontrivial, i.e., \(|Z(G)| > 1\). (See text, Th. 7.2.8, p. 328.)
   
   (a) Prove that the \(p\)-group \(G\) has a normal subgroup \(N\) with \(|N| = p\).
   
   (b) Prove that if \(|G| = p^n\) then \(G\) has normal subgroups \(N_1, N_2, \ldots, N_n\) such that \(N_1 \subseteq N_2 \subseteq \ldots \subseteq N_n\) and \(|N_i| = p^i\) for each \(i\).

2. Let \(G\) be a group (possibly infinite). Let \(N\) be a normal subgroup of \(G\) and \(H\) a subgroup of \(G\) with \(N \subseteq H \subseteq G\). Suppose \(|G:H| < \infty\). The goal of this problem is to prove that
   \[|G/N:H/N| = |G:H|\]  
   (*)

   (a) Prove (*) if \(|G| < \infty\) by using Lagrange’s Theorem.
   
   (b) Prove (*) if \(H\) is a normal subgroup of \(G\) using the Second Isomorphism Theorem.
   
   (c) Now prove (*) in general, assuming only that \(|G:H| < \infty\). (Parts (a) and (b) won’t help here.)