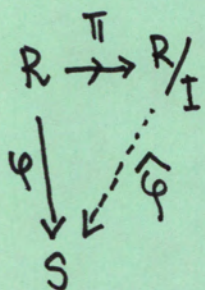


I: Let R be a commutative ring with 1 and $I \subseteq R$ an ideal ~~in~~ in R . Let

$$\pi: R \longrightarrow R/I, \quad \pi(x) = \hat{x} := x + I$$

be the canonical surjective ring morphism.

Let $\varphi: R \longrightarrow S$ be a morphism of commutative rings. Then the following hold:



1) $\exists \hat{\varphi}: R/I \longrightarrow S$ ring morphism, such that $\hat{\varphi} \circ \pi = \varphi$ if and only if

$$I \subseteq \ker \varphi.$$

2) If $\hat{\varphi}$ as above exists, then it is unique and it is given by

$$\hat{\varphi}(\hat{x}) = \varphi(x), \quad \forall x \in R.$$

Assume that $\hat{\varphi}$ as above exists. Then

3) $\hat{\varphi}$ is injective if and only if $\ker \varphi = I$.

4) $\hat{\varphi}$ is surjective if and only if φ is surjective.

II. Let ~~$R \xrightarrow{\varphi} R'$~~ $R \xrightarrow{\varphi} R'$ be a morphism of commutative rings with 1. Let

$$R \xrightarrow{i} R[x]$$

be the usual ring morphism taking $a \in R$ to the constant polynomial $a \in R[x]$.

Let $a \in R'$. Then:

1) There exists a unique ring morphism

$$R[x] \xrightarrow{\varphi_a} R'$$

with the properties that

$$\varphi_a \circ i = \varphi$$

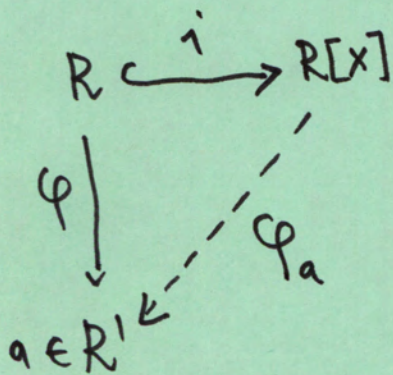
$$\varphi_a(x) = a$$

2) φ_a is given by

$$\varphi_a(a_0 + a_1x + \dots + a_nx^n) =$$

$$= \varphi(a_0) + \varphi(a_1)a + \dots + \varphi(a_n)a^n,$$

for all $(a_0 + \dots + a_nx^n) \in R[x]$.



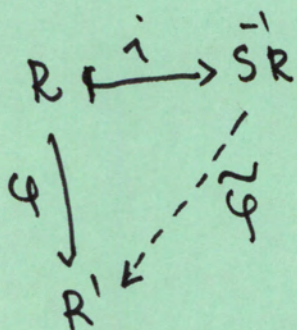
III.

Let R be a commutative ring with 1 and $S \subseteq R$ be a multiplicatively closed subset, such that $1 \in S$.

Let $i: R \rightarrow S^{-1}R$ be the canonical ring morphism, given by

$$i(r) = \frac{r}{1}, \quad \forall r \in R.$$

Let $\varphi: R \rightarrow R'$ be a morphism of commutative rings with 1 .



Then :

1) $\exists \tilde{\varphi}: S^{-1}R \rightarrow R'$ morphism of rings such that $\tilde{\varphi} \circ i = \varphi$ if and only if $\varphi(s) \in R'^{\times}$.

2) If $\tilde{\varphi}$ as above exists, then it is unique and it is given by:

$$\tilde{\varphi}\left(\frac{r}{s}\right) = \varphi(r)\varphi(s)^{-1},$$

for all $r \in R, s \in S$.
