A NOTE ON JACQUET FUNCTORS AND ORDINARY PARTS

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ABSTRACT. In this note we relate Emerton's Jacquet functor J_P to his ordinary parts functor Ord_P, by computing the χ-eigenspaces Ord_P^{χ} for central characters χ. This fills a small gap in the literature. One consequence is a weak adjunction property for *unitary* characters χ appearing in J_P, with potential applications to local-global compatibility in the p-adic Langlands program in the ordinary case.

CONTENTS

1. INTRODUCTION

In two seminal papers [Em06a] and [Em06b], Emerton introduced and studied a variant of the classical Jacquet functor (for smooth representations) in a locally analytic context, with applications to eigenvarieties in mind. Unfortunately, in this generality the functor J_P does not have as nice adjointness properties as one could hope for ($P = MN$ is a parabolic subgroup of a p-adic reductive group G). For smooth representations, J_P is a left adjoint of parabolic induction Ind_P^G, and a right adjoint of $\mathrm{Ind}_{\bar{P}}^G$ (we are ignoring a twist by the modulus character δ_P for the sake of exposition). This is no longer true in general in the locally analytic world, cf. Theorem 0.13 in [Em06b], which gives a partial result for so-called "balanced" maps, replacing $\mathrm{Ind}_{\bar{P}}^G$ by a certain subfunctor. Along the same lines, but simpler in many respects, Emerton developed the theory of ordinary parts in [Em10a] and [Em10b], thus defining a functor Ord_P which *is* right adjoint to $\mathrm{Ind}_{\bar{P}}^G$, but defined on a different category of continuous representations. In this note we relate the two functors J_P and Ord_P, see (1.2) below, and point out a weak adjunction property for *unitary* characters occurring in J_P, when P is a Borel subgroup. (Theorem 1.1.) We believe this has applications to local-global compatibility in the p-adic Langlands program for ordinary representations, cf. [BH15].

To give the flavor of the results proved in this note, we emphasize one key outcome, which we find particularly useful. The context is the following. We let G be a p-adic reductive group, which we assume is quasi-split (over $\mathbb{Q}_p$), and choose a Borel subgroup $B = TN$ with opposite $\bar{B}$ with respect to T (that is $B \cap \bar{B} = T$). Fix a finite extension $E/\mathbb{Q}_p$, and let U be a unitary Banach representation of G over E (that is, a Banach E-space U with a linear continuous action $G \times U \to U$ such that the topology on U can be defined by a G-invariant norm). Let U^{an} be the subspace of locally analytic vectors, and $J_B(U^{an})$ its Jacquet module – as defined by Emerton. The following result is then a special case of Theorem 6.2 in the main text (see also Corollary 6.4).

Theorem 1.1. *Let $\chi : T \to E^\times$ be a continuous (hence locally analytic) character, which is* **unitary** *(that is, takes values in $\mathcal{O}_E^\times$). Suppose χ occurs in $J_B(U^{an})$; meaning the χ-eigenspace $J_B^\chi(U^{an})$ is nonzero. Then there exists a nonzero, continuous, G-equivariant map*[1] $\left(Ind_{\bar{B}}^G \chi\right)^{\mathcal{C}} \longrightarrow U$.

The proof employs Emerton's theory of ordinary parts, developed in [Em10a] and [Em10b], which (as mentioned) gives a right adjoint Ord_P to continuous parabolic induction $\mathrm{Ind}_{\bar{P}}^G$, for any parabolic subgroup $P = MN$. Our main result, the aforementioned Theorem 6.2, is the existence of an M-equivariant inclusion

$$J_P^\chi(U^{an}) \hookrightarrow \mathrm{Ord}_P^\chi(U^\circ)[1/p]^{an}, \tag{1.2}$$

where $U^\circ \subset U$ is a choice of a G-stable unit ball, and $\chi : Z_M \to \mathcal{O}_E^\times$ is an arbitrary continuous character. The proof of (1.2) proceeds by making both the source and the target explicit. In [Em06a], one finds that $J_P^\chi(U^{an}) \xrightarrow{\sim} (U^{an})^{N_0, Z_M^+ = \chi}$, where $N_0 \subset N$ is an open subgroup, and $Z_M^+ = \{z \in Z_M : zN_0z^{-1} \subset N_0\}$ is the contracting central monoid (acting via double coset operators). The analogous result for Ord_P seems to be missing from the literature, although it is certainly not deep, and the argument is a simple combination of ideas from [Em06a] and [Em10a]. With future applications in mind, we feel it will be useful to have this result separately available in the literature, though. In some sense the key point is Corollary 5.1, which essentially shows that $\mathrm{Ord}_P^\chi(U^\circ) \xrightarrow{\sim} (U^\circ)^{N_0, Z_M^+ = \chi}$ (in a more general situation, allowing complete local Noetherian $\mathcal{O}$-algebras as coefficients). This easily reduces to the case of smooth representations, which in turn requires a detailed understanding of a right adjoint to the forgetful functor from smooth

[1] The superscript $\mathcal{C}$ means we take the *continuous* induction of χ, with the natural sup-norm.

M-modules to M^+-modules, where $M^+ = \{m \in M : mN_0m^{-1} \subset N_0\}$ is the (full) contracting monoid.

To motivate Theorem 1.1, we briefly outline one of the applications we have in mind. Let $\mathcal{G}_{/\mathbb{Q}}$ be a unitary group such that $\mathcal{G}(\mathbb{R})$ is compact and $G := \mathcal{G}(\mathbb{Q}_p) = \mathrm{GL}_n(\mathbb{Q}_p)$. As our U, we will take subspaces of the space of p-adic automorphic forms on $\mathcal{G}$ of tame level K^p. That is,

$$\hat{S}(K^p, E) = \{\mathcal{G}(\mathbb{Q})\backslash\mathcal{G}(\mathbb{A}_f)/K^p \xrightarrow{f} E\}^{\mathcal{C}}.$$

This is a module over a certain commutative Hecke algebra $\mathbb{T}(K^p)$, and it carries a unitary G-action. The eigenvariety $Y(K^p)$ is a rigid analytic variety parametrizing Hecke eigensystems $\mathbb{T}(K^p) \to E$ appearing in $\hat{S}(K^p, E)$. More precisely, at each point $x \in Y(K^p)$ we have an eigensystem with kernel $\mathfrak{p}_x \subset \mathbb{T}(K^p)$ say, and a character $\chi_x : T_{\mathrm{GL}_n}(\mathbb{Q}_p) \to E^\times$ occurring in the Jacquet module $J_B(\hat{S}(K^p, E)[\mathfrak{p}_x]^{an})$. In 1.1 we take $U := \hat{S}(K^p, E)[\mathfrak{p}_x]$, and deduce that in case χ_x is **unitary**,

$$\mathrm{Hom}_G^{ct}(\left(\mathrm{Ind}_B^G \chi_x\right)^{\mathcal{C}}, \hat{S}(K^p, E)[\mathfrak{p}_x]) \neq 0. \tag{1.3}$$

That is, we get intertwining operators for points x in what one might call the *ordinary* part of $Y(K^p)$. The result (1.3) should be viewed as a first step towards proving the local-global compatibility conjecture (4.2.2) of [BH15] at points x which are *not* necessarily classical.

We end this introduction with a few words on the structure of this note. In section 2, we define explicitly a right adjoint F to the forgetful functor from smooth M-modules to M^+-modules, and point out its universal property. Its importance stems from the fact that $\mathrm{Ord}_P(V) = F(V^{N_0})$, for smooth P-representations V. Section 3 defines χ-eigenspaces, for characters χ of Z_M, and gives a canonical isomorphism $F(W)^\chi \xrightarrow{\sim} W^\chi$ for smooth M^+-modules W (Proposition 3.1). In section 4 we specialize to $W = V^{N_0}$, we recall the Hecke action of M^+ on it, and compute $\mathrm{Ord}_P^\chi(V)$ for smooth V. Section 5 extends this computation to continuous representations V, by taking inverse limits. Finally, section 6 makes the comparison with the Jacquet-Emerton functor J_P, and deduces Theorem 1.1 above.

1.1. **Notation.** Throughout this note we employ the notation from [Em10a] and [Em10b]. Thus G is a p-adic reductive group (the $\mathbb{Q}_p$-points of a connected reductive group over $\mathbb{Q}_p$), and $P = MN$ is a fixed parabolic subgroup.

We let $E/\mathbb{Q}_p$ be a fixed finite extension, with integers $\mathcal{O}$, uniformizer ϖ, and residue field $\mathbb{F}$. Then $\mathrm{Comp}(\mathcal{O})$ denotes the category of complete local Noetherian $\mathcal{O}$-algebras A with finite residue field $A/\mathfrak{m}$, and $\mathrm{Art}(\mathcal{O})$ is the full subcategory of

Artinian algebras. Note that any $A \in \mathrm{Comp}(\mathcal{O})$ can be written as an inverse limit $A = \varprojlim A/\mathfrak{m}^i$ of algebras $A/\mathfrak{m}^i \in \mathrm{Art}(\mathcal{O})$.

The category of smooth G-representations V over $A \in \mathrm{Comp}(\mathcal{O})$ is denoted by $\mathrm{Mod}_G^{sm}(A)$. Recall that $v \in V$ is smooth if v is fixed by an open subgroup of G *and* annihilated by a power of $\mathfrak{m}$ (cf. Def. 2.2.1 in [Em10a]). As in Section 2.4 of loc. cit., $\mathrm{Mod}_G^{\varpi-\mathrm{cont}}(A)$ denotes the category of ϖ-adically continuous G-representations V over A. That is, those $V = \varprojlim_i V/\varpi^i V$ with $\mathcal{O}$-torsion $V[\varpi^\infty] = V[\varpi^i]$ for $i >> 0$ for which the two actions $G \times V \to V$ and $A \times V \to V$ are continuous (for the ϖ-adic topology on V). Here $V[\varpi^i] = \{v \in V : \varpi^i v = 0\}$, and $V[\varpi^\infty] = \bigcup_{i \geq 1} V[\varpi^i]$ consists of those v for which $\varpi^i v = 0$ for sufficiently large i. Note that (since $\mathcal{O}$ is a PID) V is flat over $\mathcal{O}$ if and only if V is $\mathcal{O}$-torsion free (i.e. $V[\varpi^\infty] = 0$).

2. A right adjoint to the forgetful functor

We fix a compact open subgroup $P_0 \subset P$. Let $M_0 := M \cap P_0$ and $N_0 := N \cap P_0$. Once and for all, we fix an algebra $A \in \mathrm{Comp}(\mathcal{O})$. We will restrict $A[M]$-modules to $A[M^+]$-modules, where M^+ is the contracting monoid

$$M^+ := \{m \in M : mN_0m^{-1} \subset N_0\}.$$

Introduce the (commutative) submonoid $Z_M^+ := M^+ \cap Z_M$, where Z_M is the center of M. We let $\mathrm{Mod}_{M^+}^{sm}(A)$ be the category of $A[M^+]$-modules which are smooth as M_0-representations. In this section we define a right adjoint to the forgetful functor

$$\mathrm{Mod}_M^{sm}(A) \longrightarrow \mathrm{Mod}_{M^+}^{sm}(A).$$

Actually we will have to restrict this functor to the locally Z_M-finite representations. Recall that, if V is an $A[M]$-module, we say that $v \in V$ is locally Z_M-finite if $A[Z_M] \cdot v$ is finitely generated over A (cf. Def. 2.3.1 in [Em10a]). One denotes by $V_{Z_M-\mathrm{fin}}$ the $A[M]$-submodule of locally Z_M-finite elements of V.

Definition 2.1. For an object W of $\mathrm{Mod}_{M^+}^{sm}(A)$, we let

$$F(W) := \mathrm{Hom}_{A[Z_M^+]}(A[Z_M], W)_{Z_M-\mathrm{fin}}.$$

A priori $F(W)$ is an $A[Z_M]$-module, but as [Em10a, Lemma 3.1.7(1)] shows, the Z_M-action extends naturally to an M-action via the restriction isomorphism

$$\mathrm{Hom}_{A[M^+]}(A[M], W) \xrightarrow{\sim} \mathrm{Hom}_{A[Z_M^+]}(A[Z_M], W).$$

Moreover, the resulting M-action on $F(W)$ is *smooth* (which is the reason we only take locally Z_M-finite elements in definition 2.1), cf. [Em10a, Lemma 3.1.7(2)]. Note that $F(W)$ comes equipped with a natural M^+-equivariant map

$$e : F(W) \longrightarrow W, \qquad \phi \mapsto \phi(1_{Z_M}).$$

The following observation shows that $(F(W), e)$ is a final object in the category of pairs (V, ϵ), where V is a locally Z_M-finite object of $\mathrm{Mod}_M^{sm}(A)$ and $\epsilon : V \to W$ is an M^+-equivariant map.

Proposition 2.2. *Let V be a locally Z_M-finite object of $Mod_M^{sm}(A)$ endowed with an M^+-equivariant map $\epsilon : V \to W$. Then there is a unique M-equivariant map $\hat{\epsilon} : V \to F(W)$ such that $\epsilon = e \circ \hat{\epsilon}$. In other words, $e \circ (-)$ defines a bijection*

$$Hom_{A[M]}(V, F(W)) \xrightarrow{\sim} Hom_{A[M^+]}(V, W).$$

Proof. The uniqueness of $\hat{\epsilon}$ is clear; we must have $\hat{\epsilon}(v)(z) = (z \cdot \hat{\epsilon}(v))(1_{Z_M}) = \hat{\epsilon}(zv)(1_{Z_M}) = e(\hat{\epsilon}(zv)) = \epsilon(zv)$ for arbitrary $v \in V$ and $z \in Z_M$. Conversely, define $\hat{\epsilon}(v) : A[M] \to W$ by sending $m \mapsto \epsilon(mv)$ (and extend it A-linearly). Obviously this $\hat{\epsilon}(v)$ is $A[M^+]$-linear (since ϵ is M^+-equivariant), so it defines an element of $\mathrm{Hom}_{A[M^+]}(A[M], W)$. In fact $\hat{\epsilon}(v)$ is locally Z_M-finite, so it lies in $F(W)$: Indeed $A[Z_M] \cdot v$ is finitely generated over A, by $\{v_1, \dots, v_r\}$ say. Thus $A[Z_M] \cdot \hat{\epsilon}(v)$ is generated by $\{\hat{\epsilon}(v_1), \dots, \hat{\epsilon}(v_r)\}$ over A. Finally one checks easily that the resulting map $\hat{\epsilon} : V \to F(W)$ is M-equivariant and satisfies $\epsilon = e \circ \hat{\epsilon}$. $\square$

Corollary 2.3. *The functor F is a right adjoint to the forgetful functor*

$$Mod_M^{sm}(A)_{Z_M-fin} \longrightarrow Mod_{M^+}^{sm}(A).$$

Note that (locally) admissible representations are locally Z_M-finite; by Lemma 2.3.4 in [Em10a].

3. Eigenspaces for the action of the center

For an object V of $\mathrm{Mod}_M^{sm}(A)$, and a character $\chi : Z_M \to A^\times$, we let V^χ denote the χ-eigenspace. (That is, the set of $v \in V$ such that $zv = \chi(z)v$ for all $z \in Z_M$.) Thus V^χ is a smooth $A[M]$-submodule of V, which is obviously locally Z_M-finite. Similarly, for an object W of $\mathrm{Mod}_{M^+}^{sm}(A)$, let $W^\chi = W^{Z_M^+ = \chi}$ denote the set of $w \in W$ such that $zw = \chi(z)w$ for all $z \in Z_M^+$. We let the whole center Z_M act on W^χ via χ. Observe that this Z_M-action is compatible with the M^+-action restricted to Z_M^+, and W^χ therefore acquires an action of $M = M^+ \times_{Z_M^+} Z_M$ (cf. Prop. 3.3.6 in [Em06a]). Note that W^χ is an object of $\mathrm{Mod}_M^{sm}(A)_{Z_M-\mathrm{fin}}$.

Proposition 3.1. *For any object W of $Mod_{M^+}^{sm}(A)$, and any character $\chi : Z_M \to A^\times$, the evaluation map e induces an isomorphism of smooth M-representations over A,*

$$e : F(W)^\chi \xrightarrow{\sim} W^\chi.$$

Proof. Clearly e restricts to a map $F(W)^\chi \to W^\chi$, which is equivariant for the actions of both M^+ and Z_M, which generate M. On the other hand, by the universal property of F in Proposition 2.2, the inclusion $\iota : W^\chi \hookrightarrow W$ lifts uniquely to an M-equivariant map $\hat{\iota} : W^\chi \to F(W)$, which necessarily takes values in $F(W)^\chi$. We have $e \circ \hat{\iota} = \mathrm{Id}_{W^\chi}$, by definition of $\hat{\iota}$. Moreover, unwinding definitions, one easily checks that $\hat{\iota} \circ e = \mathrm{Id}_{F(W)^\chi}$ – which essentially boils down to the observation that $\phi(z) = (z \cdot \phi)(1) = \chi(z)e(\phi)$, for $z \in Z_M$ and $\phi \in F(W)^\chi$. □

4. Eigenspaces of ordinary parts: The smooth case

We change notation a bit, and let V denote a smooth representation of P over A. Typically V will be an object of $\mathrm{Mod}_G^{sm}(A)$, but only the P-action is relevant for the definition of the ordinary part, which we briefly recall: M^+ acts on the N_0-invariants V^{N_0} via *Hecke* operators. More precisely, $m \in M^+$ acts via the double coset operator $h_{N_0,m} : V^{N_0} \to V^{N_0}$ defined by

$$h_{N_0,m}(v) = [N_0 m N_0](v) = \sum\nolimits_{n \in N_0/mN_0m^{-1}} nmv.$$

This does indeed define an action of M^+ (cf. Lemma 3.1.4 in [Em10a]), which extends the (given) M_0-action. We thus regard V^{N_0} as a smooth $A[M^+]$-module, which will serve as our "W" in the previous discussion. Following Def. 3.1.9 in loc. cit.,

$$\mathrm{Ord}_P(V) := F(V^{N_0}) = \mathrm{Hom}_{A[Z_M^+]}(A[Z_M], V^{N_0})_{Z_M-\mathrm{fin}}.$$

This is an object of $\mathrm{Mod}_M^{sm}(A)$, referred to as the P-ordinary part of V. The evaluation map $e : \mathrm{Ord}_P(V) \to V^{N_0}$ is called the canonical lifting; it is known to be an embedding for admissible V (cf. Theorem 3.3.3 in loc. cit.), but we will not use that. In what follows we will write $\mathrm{Ord}_P^\chi(V)$ instead of $\mathrm{Ord}_P(V)^\chi$, for central characters χ. The following result is an immediate consequence of Proposition 3.1.

Corollary 4.1. *For any object V of $Mod_P^{sm}(A)$, and any character $\chi : Z_M \to A^\times$, the canonical lifting induces an isomorphism of smooth M-representations over A,*

$$e : Ord_P^\chi(V) \xrightarrow{\sim} V^{N_0, Z_M^+ = \chi}.$$

In the next section we will extend this Corollary to *continuous* representations.

5. Eigenspaces of ordinary parts: The general case

In Section 3.4 of [Em10a], Emerton extends Ord_P to the category of ϖ-adically continuous representations. For an object V of $\mathrm{Mod}_P^{\varpi-\mathrm{cont}}(A)$ (cf. Section 1.1 in the Introduction),

$$\mathrm{Ord}_P(V) := \varprojlim_i \mathrm{Ord}_P(V/\varpi^i V).$$

Note that each $V/\varpi^i V$ is an $\mathcal{O}$-torsion object of $\mathrm{Mod}_P^{sm}(A)$; smoothness follows from the continuity of the actions of G and A. By Prop. 3.4.6 of loc. cit., $\mathrm{Ord}_P(V)$ is an object of $\mathrm{Mod}_M^{\varpi-\mathrm{cont}}(A)$, equipped with a canonical lifting $e : \mathrm{Ord}_P(V) \to V^{N_0}$. The χ-eigenspaces are defined as in Section 3.

Corollary 5.1. *For any object V of $Mod_P^{\varpi-cont}(A)$, and any character $\chi : Z_M \to A^\times$, the canonical lifting induces an isomorphism in $Mod_M^{\varpi-cont}(A)$,*

$$e : Ord_P^\chi(V) \xrightarrow{\sim} V^{N_0, Z_M^+ = \chi}.$$

Proof. This is a formal consequence of the smooth case (Corollary 4.1). Indeed,

$$\begin{aligned} \mathrm{Ord}_P^\chi(V) &= \varprojlim_i \mathrm{Ord}_P^\chi(V/\varpi^i V) \\ &\simeq \varprojlim_i \Big((V/\varpi^i V)^{N_0, Z_M^+ = \chi} \Big) \\ &= \Big(\varprojlim_i V/\varpi^i V \Big)^{N_0, Z_M^+ = \chi} \\ &= V^{N_0, Z_M^+ = \chi}, \end{aligned} \tag{5.2}$$

as desired. ☐

6. The relation to the Jacquet-Emerton functor

In this section we take $A = \mathcal{O}$, and consider an object V of $\mathrm{Mod}_G^{\varpi-\mathrm{cont}}(\mathcal{O})$ which is flat over $\mathcal{O}$ (i.e., $\mathcal{O}$-torsion free: $V[\varpi^\infty] = \bigcup_{i=1}^\infty V[\varpi^i] = 0$). Then $V[1/p] := V \otimes_{\mathbb{Z}_p} \mathbb{Q}_p = V \otimes_{\mathcal{O}} E$ becomes a Banach space over E, equipped with a norm $\|\cdot\|$ for which V is the unit ball. Thus $V[1/p]$ becomes a unitary Banach representation of G over E, and we may take its subspace of locally analytic vectors $V[1/p]^{an}$ (cf. Chapter 7 of [ST03]) which is a locally analytic G-representation (of compact type). We apply Emerton's Jacquet functor J_P from [Em06a], and consider $J_P(V[1/p]^{an})$; a locally analytic M-representation. If $\chi : Z_M \to E^\times$ is a locally analytic character, Proposition 3.4.9 of loc. cit. shows that

$$J_P^\chi(V[1/p]^{an}) \xrightarrow{\sim} \Big(V[1/p]^{an} \Big)^{N_0, Z_M^+ = \chi}. \tag{6.1}$$

Recall that $J_P^\chi(U^{an})$ denotes the (closed) subrepresentation of $J_P(U^{an})$ on which Z_M acts through the character χ; the χ-eigenspace, cf. the bottom line of page 35 in [Em06a].

Theorem 6.2. *Let V be an object of $Mod_G^{\varpi-cont}(\mathcal{O})$, which is flat as an $\mathcal{O}$-module. Let $\chi : Z_M \to \mathcal{O}^\times$ be a unitary continuous character (which is automatically locally analytic). Then there is an M-equivariant embedding*

$$J_P^\chi(V[1/p]^{an}) \hookrightarrow Ord_P^\chi(V)[1/p]^{an}. \tag{6.3}$$

Proof. From Corollary 5.1 we know that $\mathrm{Ord}_P^{\chi}(V) \xrightarrow{\sim} V^{N_0, Z_M^+ = \chi}$. Inverting p,

$$\mathrm{Ord}_P^{\chi}(V)[1/p] \simeq \Big(V^{N_0, Z_M^+ = \chi}\Big) \otimes_{\mathcal{O}} E = \Big(V \otimes_{\mathcal{O}} E\Big)^{N_0, Z_M^+ = \chi} = V[1/p]^{N_0, Z_M^+ = \chi}.$$

(For the second step note that vectors of $V \otimes_{\mathcal{O}} E$ are pure tensors $v \otimes \varpi^i$, and the map $v \mapsto v \otimes \varpi^i$ is injective.) This clearly contains the right-hand side of (6.1). Moreover, if $x \in V[1/p]^{N_0, Z_M^+ = \chi}$ is a vector such that the orbit map $f_x : G \to V[1/p]$ given by $g \mapsto gx$ is locally analytic (i.e., x is in the right-hand side of (6.1)), then the orbit map $\tilde{f}_x : M \to V[1/p]^{N_0, Z_M^+ = \chi}$ given by $m \mapsto mx$ is also locally analytic: Recall that M^+ acts via the double coset operators $h_{N_0,m}$, which are given by finite sums; $\tilde{f}_x(m) = \sum_{n \in N_0/mN_0m^{-1}} f_x(nm)$ for $m \in M^+$. The sum extends over the set N_0/mN_0m^{-1} which appears to depend on m. However, $m_0 N_0 m_0^{-1} = N_0$ for $m_0 \in M_0$ (recall the definitions $N_0 = N \cap P_0$ and $M_0 = M \cap P_0$ in the first sentence of section 2), so at least on the open neighborhood mM_0 the sum defining $\tilde{f}_x$ extends over a *fixed* finite set of coset representatives $\{n\} \subset N_0$, independent of m. We conclude that $\tilde{f}_x = \sum_n \ell_{n^{-1}} f_x$ on the open neighborhood mM_0 and hence $\tilde{f}_x$ is locally analytic there. Finally we note that $M = M^+ \times_{Z_M^+} Z_M$ (cf. [Em06a, 3.3.6]) and the action of Z_M on x just comes from the G-action on V. Therefore $\tilde{f}_x$ is indeed locally analytic on all of M as claimed. □

The converse does not seem to hold (that $\tilde{f}_x$ in the above proof is locally analytic *only* if f_x is locally analytic), so (6.3) is not expected to be surjective in general.

We single out the following immediate consequence of Theorem 6.2.

Corollary 6.4. *Let V be an object of $Mod_G^{\varpi-cont}(\mathcal{O})$ with $V[\varpi^\infty] = 0$ (i.e., which is flat over $\mathcal{O}$), and let $\chi : Z_M \to \mathcal{O}^\times$ be a unitary continuous character occurring in $J_P(V[1/p]^{an})$. Then*

$$Ord_P^{\chi}(V) \neq 0.$$

In the special case where P is a Borel subgroup (so $M = Z_M$ is a maximal torus), we can combine the previous Corollary with the main adjunction formula for Ord_P (cf. Theorem 4.4.6 in [Em10a]) and derive that, for unitary $\chi \hookrightarrow J_P(V[1/p]^{an})$,

$$\mathrm{Hom}_G(\mathrm{Ind}_{\bar{P}}^G \chi, V) \neq 0, \tag{6.5}$$

where $\bar{P}$ is the opposite parabolic of P with respect to M (characterized by the identity $P \cap \bar{P} = M$) and $\mathrm{Ind}_{\bar{P}}^G$ is continuous parabolic induction (as introduced in section 4.1 of [Em10a]).

We envision that (6.5) will have applications to local-global compatibility in the p-adic Langlands program in the context of ordinary representations (cf. Conjecture

4.2.2 in [BH15]), where one would take $V[1/p]$ to be a certain Banach space of p-adic modular forms $\hat{S}(K^p, E)$ on a definite unitary group (where K^p is the tame level). For more details we refer the reader to the introduction.

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