#34. Prove \((ab)^2 = a^2b^2 \iff ab = ba\).

**Pf:**
\[(ab)^2 = abab = a^2b^2 \iff bab = ab \iff ba = ab.\]

#38. Consider \(D_4\), the dihedral group of order 8.
Then, \(SRS = r^{-1}\) and \(r^{-1} = r^3\), so
\[SRS = r^3 = rrr\]
but \(r^2 \neq r^2\) 
\[1'\]

#46. Prove that \(G = \{3^m6^n : m,n \in \mathbb{Z}\}\) is a group under multiplication.

**Pf:**
- Obviously, \(G \neq \emptyset\).
- \((3^{m_1}6^{n_1})(3^{m_2}6^{n_2}) = 3^{m_1+m_2}6^{n_1+n_2} \in G\) so \(G\) is closed.
- \(m = n = 0\) gives \(1 \in G\) identity.
- \((3^m6^n)(3^{-m}6^{-n}) = 1\) hence inverses.
- \(3^{m_1}6^{n_1}(3^{m_2}6^{n_2})(3^{m_3}6^{n_3})) = 3^{m_1}6^{n_1}(3^{m_2+m_3}6^{n_2+n_3}) = 3^{m_1+(m_2+m_3)}6^{n_1+(n_2+n_3)} = 3^{(m_1+m_2)+m_3}6^{(n_1+n_2)+n_3} = (3^{m_1}6^{n_1})(3^{m_2}6^{n_2})(3^{m_3}6^{n_3})\)
hence associativity. 
\[1''\]
G = \{ (aa) : a \in \mathbb{R} \setminus \{0\} \}, show G is a group under matrix multiplication.

Clearly G \neq \emptyset.

Indeed, \((aa)(b b) = (2ab 2ab) \in G\) so closed.

Identity is \((\frac{1}{2} \frac{1}{2})\) since \((aa)(\frac{1}{2} \frac{1}{2}) = (aa)\) and \((\frac{1}{2} \frac{1}{2})(aa) = (aa)\).

Inverse of \((aa)\) is \((\frac{1}{2a} \frac{1}{2a})\).

Associativity is clear since \(G \subseteq \mathcal{M}_2(\mathbb{R})\).

Each element has an inverse in \(G\) because the identity element of \(G\) is not \((1 0)\) which is the usual "identity" in matrix groups.

\((G, +)\) elements of \(<\frac{1}{2}>\) are \(\{ \frac{m}{2} : m \in \mathbb{Z} \}\).

\((\mathbb{Q}_2, \cdot)\) elements of \(<\frac{1}{2}>\) are \(\{ \frac{1}{2m} : m \in \mathbb{Z} \}\).

If \(G\) group, \(x \in G\), then \(\text{order}(x) = \text{order}(x^{-1})\).

Pf: let \(m = \text{order}(x)\), then

\((x^{-1})^m = x^{-m} = (x^m)^{-1} = e^{-1} = e\)

\(\implies \text{order}(x^{-1}) \mid m\).

Let \(n = \text{order}(x^{-1})\), then \(\exists q, r \in \mathbb{Z}\) s.t. \(n = mq + r\) with \(0 \leq r < m\). Then

\(e = (x^{-1})^n = (x^n)^{-1} = (x^{mq+r})^{-1} = (x^r)^{-1} = x^{-r}\)

so, \(r = 0 \implies \text{order}(x^{-1}) = \text{order}(x)\).

\[ \text{hence } \quad U(4) = \langle [3] \rangle \]

Similarly, for \( U(11) = \langle [10] \rangle \).

Similarly, for \( U(11) = \langle [3] \rangle \).

\[ \text{hence } \quad U(11) = \langle [11] \rangle \]

\[ U(11) \neq \langle [11] \rangle \]

\[ H, K \leq G \text{ show } \text{H} \cap \text{K} \leq G. \]

If \( \{ \text{H}_i \}_{i \in I} \) be a set of subgroups of G.

For \( x, y \in \text{H}_i \), \( x, y \in \text{H}_j \) \( \forall i, j \in I \)

\[ \text{G}\text{iven } x \in \text{H}_i, \quad x \text{H}_i \leq G \quad \text{H}_i \text{H}_j \]

\[ x \text{H}_i \leq \text{H}_i \text{H}_j \]

\[ x \in \text{H}_i \text{H}_j \]

\[ x \in \text{H}_i \text{H}_j \]

\[ \text{Just for some } \]

\[ \sum_{i=1}^{n} a_i \]

\[ \#32 \]

\[ \#30 \]
In \( \mathbb{Z} \) find the following.

a.) \( \langle 8, 14 \rangle = \langle 2 \rangle \)
b.) \( \langle 8, 13 \rangle = \langle 1 \rangle \)
c.) \( \langle 6, 15 \rangle = \langle 3 \rangle \)
d.) \( \langle m, n \rangle = \langle (m, n) \rangle \)
e.) \( \langle 12, 18, 45 \rangle = \langle 3 \rangle \)