Math 103A, Fall 2017

Modern Algebra, HW 3

Due Tuesday October 24th by 5PM in your TA's box/after discussion session

From Lauritzen's book:

- Exercises 1.12 (starting page 41): *6 (not part (viii)), 12, 18, 29, 30*[1]

Problem A.

(a) Write down all the elements of $(\mathbb{Z}/12\mathbb{Z})^\times$ and find $\phi(12)$.

(b) Give the composition table for $(\mathbb{Z}/12\mathbb{Z})^\times$ with multiplication (cf. Definition 2.1.4 on page 53 in the book); which elements satisfy $x^2 = 1$?

Problem B.

(a) Find the integer n such that $11n \equiv 1 \pmod{13}$ and $0 \leq n < 13$.

(b) Find all integers x in the range $[0, 500]$ satisfying *both* congruences:

$$x \equiv 5 (\text{mod } 11) \qquad \underline{\text{and}} \qquad x \equiv 7 (\text{mod } 13).$$

Problem C. Let $f : X \to Y$ be a function. Define a relation $\sim$ on X by

$$x \sim x' \iff f(x) = f(x').$$

(a) Prove that $\sim$ is an equivalence relation on X.

(b) Show that the equivalence classes are the non-empty level sets ($y \in Y$);

$$f^{-1}(y) = \{x \in X : f(x) = y\}.$$

(c) Explain why *any* equivalence relation on X arises in this fashion from a suitable function f. (`Hint`: Take Y to be the set of equivalence classes.)

[1] `Hint:` In (iii) suppose $d < n$ divides n. Use the assumption with $i = d$. Conclude $d = 1$.