Due Tuesday October 24th by 5PM in your TA’s box/after discussion session

From Lauritzen’s book:

• Exercises 1.12 (starting page 41): 6 (not part (viii)), 12, 18, 29, 30

Problem A.

(a) Write down all the elements of \((\mathbb{Z}/12\mathbb{Z})^\times\) and find \(\phi(12)\).

(b) Give the composition table for \((\mathbb{Z}/12\mathbb{Z})^\times\) with multiplication (cf. Definition 2.1.4 on page 53 in the book); which elements satisfy \(x^2 = 1\)?

Problem B.

(a) Find the integer \(n\) such that \(11n \equiv 1 \pmod{13}\) and \(0 \leq n < 13\).

(b) Find all integers \(x\) in the range \([0, 500]\) satisfying both congruences:

\[x \equiv 5 \pmod{11} \quad \text{and} \quad x \equiv 7 \pmod{13}.

Problem C. Let \(f : X \to Y\) be a function. Define a relation \(\sim\) on \(X\) by

\[x \sim x' \iff f(x) = f(x').\]

(a) Prove that \(\sim\) is an equivalence relation on \(X\).

(b) Show that the equivalence classes are the non-empty level sets \((y \in Y)\):

\[f^{-1}(y) = \{x \in X : f(x) = y\}.

(c) Explain why any equivalence relation on \(X\) arises in this fashion from a suitable function \(f\). (Hint: Take \(Y\) to be the set of equivalence classes.)

\footnote{Hint: In (iii) suppose \(d < n\) divides \(n\). Use the assumption with \(i = d\). Conclude \(d = 1\).}