Due Tuesday December 5th by 5PM in your TA’s box/after discussion session

From Lauritzen’s book:

- Exercises 2.11 (starting page 104): 30, 40, 41, 44, 45

**Problem A.** Consider the following permutation in $S_9$,

$$\alpha = (1934)(3456)(6782).$$

(a) Write $\alpha$ as a composition of disjoint cycles.

(b) Find the order of $\alpha$ and $\alpha^{2017}$.

(c) Find the sign of $\alpha$; does $\alpha$ belong to $A_9$?

**Problem B.** Recall the Klein four-group from Problem A(b) on HW4:

$$V_4 = \{\pm 1\} \times \{\pm 1\} = \{(1,1),(1,-1),(-1,1),(-1,-1)\}.$$  

Let $\phi: V_4 \to S_4$ be the Cayley homomorphism (for the above ordering), which can be read off from the composition table.

(a) Factor all elements of $\text{im}(\phi)$ into disjoint cycles.

(b) Verify that $\text{im}(\phi)$ is a normal subgroup of $A_4$.

(c) Is there a surjective homomorphism $A_4 \to \mathbb{Z}/3\mathbb{Z}$?

(d) Is there a surjective homomorphism $S_4 \to \mathbb{Z}/3\mathbb{Z}$?

**Problem C.** Let $n \ge 4$. Show that the center\(^1\) of the alternating group $A_n$ only contains the identity element. In other words that

$$Z(A_n) = \{e\}.$$  

(Hint: Use the formula $\alpha(abc)\alpha^{-1} = \alpha(a)\alpha(b)\alpha(c)$ to see that $\alpha$ maps every subset $\{a,b,c\}$ to itself. What about $\{a,b\}$ and $\{a\}$? What is $Z(A_3)$?)

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\(^1\)Recall Problem D on HW6. The center consists of $\alpha$ such that $\alpha\beta = \beta\alpha$ for all $\beta$. 

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