Math 103A, Fall 2017

Modern Algebra, HW 9

Due Tuesday December 5th by 5PM in your TA's box/after discussion session

From Lauritzen's book:

- Exercises 2.11 (starting page 104): *30, 40, 41, 44, 45*

Problem A. Consider the following permutation in S_9,

$$\alpha = (1934)(3456)(6782).$$

(a) Write α as a composition of **disjoint** cycles.

(b) Find the order of α and α^{2017}.

(c) Find the sign of α; does α belong to A_9?

Problem B. Recall the Klein four-group from Problem A(b) on HW4:

$$V_4 = \{\pm 1\} \times \{\pm 1\} = \{(1,1), (1,-1), (-1,1), (-1,-1)\}.$$

Let $\phi : V_4 \to S_4$ be the Cayley homomorphism (for the above ordering), which can be read off from the composition table.

(a) Factor all elements of $\mathrm{im}(\phi)$ into disjoint cycles.

(b) Verify that $\mathrm{im}(\phi)$ is a normal subgroup of A_4.

(c) Is there a surjective homomorphism $A_4 \to \mathbb{Z}/3\mathbb{Z}$?

(d) Is there a surjective homomorphism $S_4 \to \mathbb{Z}/3\mathbb{Z}$?

Problem C. Let $n \geq 4$. Show that the center[1] of the alternating group A_n only contains the identity element. In other words that

$$Z(A_n) = \{e\}.$$

(`Hint`: Use the formula $\alpha(abc)\alpha^{-1} = \big(\alpha(a)\alpha(b)\alpha(c)\big)$ to see that α maps every subset $\{a, b, c\}$ to itself. What about $\{a, b\}$ and $\{a\}$?) What is $Z(A_3)$?

[1] Recall Problem D on HW6. The *center* consists of α such that $\alpha\beta = \beta\alpha$ for all β.