Math 103A, Fall 2019

Modern Algebra I, HW 1

Due Friday October 11th at 11:30AM in Ji Zeng's box outside B402A.

From Armstrong's Groups and Symmetry:

- Exercises (Chapter 2, pages 9–10):

  2.3, 2.8

- Exercises (Chapter 3, pages 13–14):

  3.1, 3.2

Problem A. Let $(G, *)$ be a set with a composition law. An element $e \in G$ is called <u>neutral</u> if the equalities $a * e = a$ and $e * a = a$ both hold for all $a \in G$.

(a) Prove that there can be at most one neutral element e.

(b) Give an example of a pair $(G, *)$ which does not admit a neutral element.

Problem B. Keep the setup in Problem A, but add the assumptions that $(G, *)$ satisfies the associative law and has a neutral element e. Given an $a \in G$, an <u>inverse</u> of a is an element $b \in G$ for which $a * b = e$ and $b * a = e$.

(a) Show that any element $a \in G$ can have at most one inverse b.

(b) Consider $\mathbb{Z}$ with multiplication. Which elements have an inverse in $\mathbb{Z}$?

Problem C. For each of the congruence equations below find all integer solutions x in the range $|x| < 25$.

(a) $x + 3 \equiv 5 \pmod{11}$

(b) $3x \equiv 5 \pmod{11}$

(c) $5x + 3 \equiv 1 \pmod{11}$

Problem D.

(a) Compute $\mathrm{GCD}(899, 1147)$ and $\mathrm{LCM}(899, 1147)$. (`Hint`: Euclid.)

(b) Find a pair of integers (x, y) satisfying the following relation:

$$\mathrm{GCD}(899, 1147) = 899x + 1147y.$$

(c) Factor each of the number 899 and 1147 into prime numbers.