LECTURE 23

(Monday Nov. 25, 2019)

$\underline{\text{EX}}$ $H = \langle (123) \rangle = \{e, (123), (132)\}$

H is normal in S_3 (idx $= 2$),

H is <u>not</u> normal in S_4:

$$\underset{\substack{\cap \\ S_4}}{(14)}\underset{\substack{\cap \\ H}}{(123)}(14)^{-1} = (423) \text{ not in } H.$$

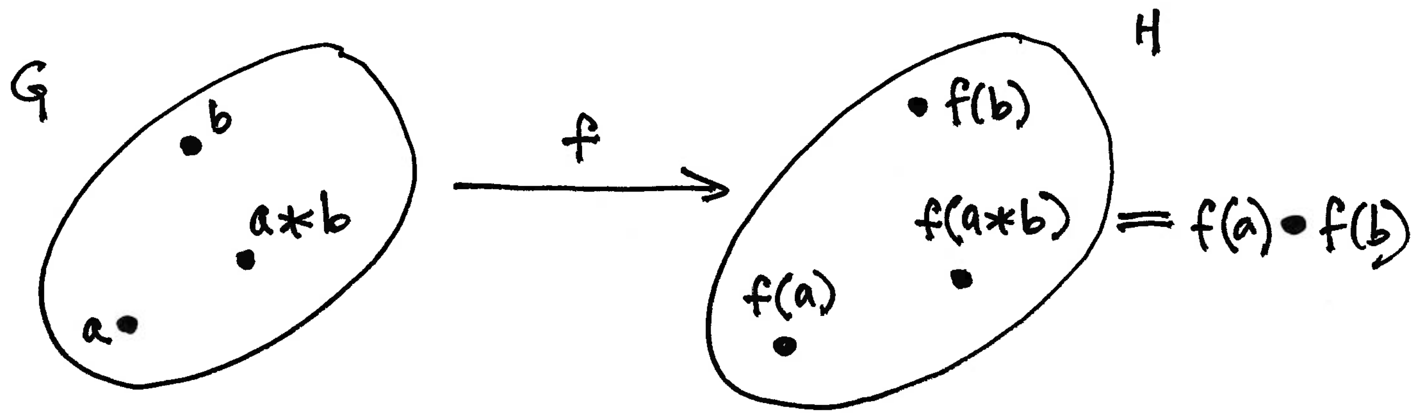
(Recall $H \leq G$ is "<u>normal</u>" iff it's <u>closed under conjugation</u>:

$a \in G \wedge h \in H \implies aha^{-1} \in H$)

"<u>Homomorphism</u>" is a function $f: G \longrightarrow H$ which is <u>compatible</u> with the composition laws — in the sense that: $\forall a, b \in G$,

$(G, *)$ and $(H, \bullet)$ any two groups.

$$f(a * b) = f(a) \bullet f(b)$$



<u>Ex</u> Always have the <u>trivial</u> homomorphism $f: G \longrightarrow H$ sending <u>any</u> $a \in G$ to $f(a) = e_H$.

<u>EXC</u> (i) $f(e_G) = f(e_H)$

(why? $f(e_G) = f(e * e) = f(e) \bullet f(e)$

$\quad \| $

$f(e) \bullet e_H \quad \xRightarrow{\text{cancellation law}} f(e_G) = e_H$.)

(ii) $f(a^{-1}) = f(a)^{-1} \quad \forall a \in G$.

(why? $f(a) \bullet f(a^{-1}) = f(a * a^{-1}) = f(e_G) \overset{(i)}{=} e_H$)