Math 103A, Modern Algebra I, MT2

Wednesday, November 13th, 2019, 10-10:50am, APM B402A

- *Your Name*:
- *ID Number*:
- *Section*:

B01 (5:00 PM) B02 (6:00 PM)

Problem #	Points (out of 10)
1	
2	
3	
4	
Total (out of 40):	

Problem 1. Let $(G, *)$ be a cyclic group of size 6. Choose a generator $a \in G$.

(a) Find the <u>order</u> of each of its elements:

$$e \qquad a \qquad a^2 \qquad a^3 \qquad a^4 \qquad a^5.$$

Circle those x above for which $G = \langle x \rangle$ holds.

(b) List the elements of the non-trivial subgroups $\langle a^2 \rangle$ and $\langle a^3 \rangle$.

(c) Is the product $G \times G$ abelian? Is $G \times G$ cyclic? If so find a generator.

Problem 2. Recall that $(\mathbb{Z}_7^\times, \bullet)$ is the multiplicative group of invertible residue classes modulo 7.

(a) Check that the residue class $[3]$ generates $\mathbb{Z}_7^\times$. Find its size $|\mathbb{Z}_7^\times|$.

(b) Find the order of each of the elements:

$$[1] \qquad [2] \qquad [3] \qquad [4] \qquad [5] \qquad [6].$$

Circle those $[x]$ above for which $\mathbb{Z}_7^\times = \langle [x] \rangle$ holds.

(c) Find the integer r in the interval $0 \leq r < 7$ satisfying the congruence

$$3^{71} \equiv r \pmod{7}.$$

Problem 3. Let D_4 be the dihedral group of symmetries of a square centered at the origin. Introduce

r=rotation by $\frac{\pi}{2}$ in the counterclockwise direction,

s=reflection across the horizontal axis.

Recall the relations $r^4 = s^2 = e$ and $rs = sr^{-1}$.

(a) Give its cardinality $|D_4|$. Is D_4 an abelian group?

(b) Describe geometrically what the transformation rs does to a point in the plane: If rs is a rotation give the angle, if rs is a reflection give the axis.

(c) Identify the element srs^{-1} on the list below. Justify your answer.

$$e \qquad r \qquad r^2 \qquad r^3 \qquad s \qquad sr \qquad sr^2 \qquad sr^3$$

Problem 4. Let $\alpha \in S_7$ be the permutation given by $\alpha = (1352)(5137)$.

(a) Express α in array form. That is, fill in the blank boxes below.

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ \square & \square & \square & \square & \square & \square & \square \end{pmatrix}$$

(b) Is α a cycle? If not, find its decomposition into disjoint cycles.

(c) Compute $\operatorname{ord}(\alpha)$ and $\operatorname{sign}(\alpha)$. Does α belong to A_7?