Math 104A, Fall 2018

Number Theory, HW 2

Due Wednedsday October 17th by 5PM in Shubham Sinha's box.

From Weissman's book `An illustrated theory of numbers`:

- Exercises (Section 1, pages 44–45):

  2, 6, 7, 9, 10, 11, 13

Problem A. Consider a monic polynomial equation with integer coeffcients

$$x^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 = 0.$$

Suppose $x = \frac{r}{s}$ is a rational root. Show that x is necessarily an *integer* which divides the constant term a_0. (`Hint`: You may assume $\text{GCD}(r, s) = 1$. Now use Euclid's lemma on page 56 after multiplying the equation by s^n.)

Problem B. Find $\text{GCD}(5183, 4757)$ and express it as a linear combination $5183x + 4757y$ for a suitable pair of integers x, y. Then find <u>all</u> pairs of integers (x, y) satisfying both conditions below.

$$\text{GCD}(5183, 4757) = 5183x + 4757y, \qquad 0 < x < 200.$$

Problem C. Recall that the sequence of Fibonacci numbers $(F_n)_{n\geq 0}$ is defined recursively by

$$F_n = F_{n-1} + F_{n-2} \qquad \forall n > 1$$

with initial values $F_0 = 0$ and $F_1 = 1$. List the first ten Fibonacci numbers.

(a) Use induction on n to prove the so-called Cassini identity –

$$F_{n+1}F_{n-1} - F_n^2 = (-1)^n \qquad \forall n > 0.$$

(b) Deduce from (a) that any two neighboring Fibonacci numbers are coprime:

$$\text{GCD}(F_{n-1}, F_n) = 1 \qquad \forall n > 0.$$