Math 104A, Fall 2018

Number Theory, HW 4

Due Wednedsday October 31st by 5PM in Shubham Sinha's box.

From Weissman's book `An illustrated theory of numbers`:

- Exercises (Section 2, pages 72–73):

  8, 11[1]*, 12, 14, 15, 16, 17*

- Exercises (Section 3, pages 96–97):

  12

Problem A.

(a) Is there a Pythagorean triple (x, y, z) with hypotenuse $z = 71$? (`Hint`: Observe that any number of the form $a^2 + b^2$ is never congruent to 3 modulo 4. Why not?)

(b) Find a Pythagorean triple (x, y, z) with hypotenuse $z = 73$.

Problem B. (This exercise establishes the converse to Problem B on HW3.) Let $p > 1$ be an integer with the property that p divides $\binom{p}{r}$ for all $0 < r < p$. Prove that p must be a prime number. (`Hint`: Otherwise pick a prime number $\ell | p$ satisfying $0 < \ell < p$. Then by assumption $p | \binom{p}{\ell}$. Now justify and use the identity

$$\ell! \cdot \frac{1}{p} \binom{p}{\ell} = (p-1)(p-2)\cdots(p-\ell+1).$$

Why can ℓ not divide any of the factors on the right?)

[1]Show that $n = p^{\ell-1}$ for primes p and ℓ (and conversely).