Math 104A, Fall 2018

Number Theory, HW 6

Due Wednesday November 14th by 5PM in Shubham Sinha's box.

From Weissman's book `An illustrated theory of numbers`:

- Exercises (Section 5, pages 150–151):

  11(a+b+c) $\longleftarrow$ not part (d)

- Exercises (Section 6, pages 170–171):

  1, 3, 6, 7, 12, 13

- Exercises (Section 7, pages 190–191):

  1, 2

Problem A. (These questions arose in class.) Let $a, b \in \mathbb{Z}$ have residue classes $[a]$ and $[b]$ modulo some fixed $m > 1$. Consider the set of all products

$$[a] \star [b] := \{xy : x \in [a] \wedge y \in [b]\}.$$

One question asked was essentially whether this defines a multiplication on $\mathbb{Z}_m$.

(a) Prove that there is always an inclusion $[a] \star [b] \subseteq [ab]$.

(b) Show that $[0] \star [0] \neq [0]$ and conclude that $[0] \star [0]$ is not a residue class modulo m (but rather a residue class modulo m^2).

(c) How about addition – is it true that

$$[a + b] \stackrel{?}{=\!=} \{x + y : x \in [a] \wedge y \in [b]\}.$$

Prove it or give a counterexample.