

Due Thursday February 28th by 10AM in Shubham Sinha's box.

From Niven, Zuckerman, Montgomery (5th Ed.):

- Problems (Section 7.1, page 327):

1, 3, 4, 5

- Problems (Section 7.3, page 333):

1, 2, 3

**Problem A.** Fix an  $a \in \mathbb{N}$ , and define a sequence of positive integers  $(x_n)_{n \geq 0}$  recursively by the equations

$$x_n = ax_{n-1} + x_{n-2} \quad x_1 = a \quad x_0 = 1$$

- (a) Give the first five terms of the sequence  $x_0, x_1, x_2, x_3, x_4$ .
- (b) Show that  $\frac{x_n}{x_{n-1}}$  has the following continued fraction  $\forall n \geq 1$ :

$$\frac{x_n}{x_{n-1}} = \langle a, \dots, a \rangle = a + \frac{1}{a + \dots + \frac{1}{a}}$$

(There are  $n$  partial quotients  $a$  on the right.)

- (c) Conclude that  $\frac{x_n}{x_{n-1}}$  converges as  $n \rightarrow \infty$  and find the limit.
- (d) Do you recognize the sequence  $(x_n)_{n \geq 0}$  for  $a = 1$ ? (**Hint:** Rabbits.)