Math 104B, Winter 2019

Number Theory II, HW 8

Due Thursday March 7th by 10AM in Shubham Sinha's box.

From Niven, Zuckerman, Montgomery (5th Ed.):

- Problems (Section 7.3, page 333):

  4, 5, 6

- Problems (Section 7.4, page 336):

  1

- Problems (Section 7.6, page 344):

  1, 2

Problem A. Let $\alpha > 1$ be an irrational number, whose convergents we denote by $c_n(\alpha)$. Similarly for $\frac{1}{\alpha}$.

(a) Show that $\forall n > 0$ we have the relation

$$c_n(\frac{1}{\alpha}) = c_{n-1}(\alpha)^{-1}.$$

(b) Write $c_n(\alpha)$ in its lowest terms $\frac{r_n(\alpha)}{s_n(\alpha)}$ (analogously for $\frac{1}{\alpha}$) and deduce the following identity from (a).

$$r_n(\frac{1}{\alpha})r_{n-1}(\alpha) = s_n(\frac{1}{\alpha})s_{n-1}(\alpha).$$

Problem B. Verify the three identities below for all integers $a > 0$.

(a) $\sqrt{a^2+1} = \langle a, \overline{2a} \rangle$

(b) $\sqrt{a^2+2} = \langle a, \overline{a, 2a} \rangle$

(c) $\sqrt{a^2-2} = \langle a-1, \overline{1, a-2, 1, 2a-2} \rangle$ – assuming $a > 2$.